Name...
I.D. number...

Math 2210-3
Exam 2
April 7, 2010

This exam is closed-book and closed-note, except for a single 3 by 5 inch card filled with whatever information you wish. You may use a scientific calculator, but not one which is capable of graphing or doing calculus (for example, you may not use a graphing calculator). **In order to receive full or partial credit on any problem, you must show all of your work** and **justify your conclusions.** There are 100 points possible. The point values for each problem are indicated in the right-hand margin. **Good Luck!**

1 ______________ (35)

2 ______________(20)

3 ______________(20)

4 ______________(10)

5 ______________(15)

Total ______________(100)

1a) Compute

$$\int_{-1}^{1}\int_{x^2}^{1} y \, dy \, dx$$

(10 points)

1b) Sketch the region of integration for the iterated integral in (1a). Label the boundary curves and the points at which they intersect.

(5 points)

1c) Set up and compute the double integral in (1a) with the order of integration reversed.

(10 points)

1d) Suppose that a laminate in the plane region you sketched for (1b) has density function $\delta(x, y) = y$, so that your integral computations in 1a) and 1c) were for the mass of the laminate. Find the center of mass of this laminate. Hint: you can deduce one of the two coordinates using symmetry.

(10 points)

2) One fine afternoon in Spring Valley the surface temperature is given by

$$U(x, y) = 50 - 2y + 3xy \quad \text{degrees,}$$

where x and y denote miles east and north of downtown, respectively.

2a) In what direction is the temperature increasing most rapidly at location (2, 1), and what is the rate of increase there? Include correct units you your answer.

(5 points)

2b) An intrepid hiker passes through the location (2, 1) that afternoon, hiking due north at 3 miles/hour. How fast is the temperature she experiences changing? Include correct units in your answer.

(5 points)

2c) If the hiker wishes instead to stay on the 54 degree isotherm, in what direction should she be going as she passes through (2,1)?

(5 points)

2d) Use the tangent (differential) approximation formula for the temperature function, with base point (2, 1), to estimate the temperature at location (2.2, 0.9).

(5 points)

3a) Sketch the triangular piece of the plane

$$x + y + 2z = 12$$

which is in the first octant.

(5 points)

3b) Use Calculus to find the point on the plane of (3a) which is closest to the origin. Note that you can do this problem by minimizing the function $x^2 + y^2 + z^2$ for points on the plane. You may use either method we discussed in class.

(15 points)

4) Consider the 1-sheeted elliptic hyperbolid with implicit equation

$$x^2 + y^2 - 2z^2 = 3.$$

Find the equation of the tangent plane to this surface at the point (2, -1, -1).

(10 points)

5a) Let B be the ball domain inside the radius-R sphere, i.e. the points for which

$$x^2 + y^2 + z^2 \leq R^2 .$$

Write an iterated triple integral in Euclidean (rectangular) coordinates which would yield the volume of this ball. Do not evaluate this integral. (The point of this problem is to see if you can figure out correct limits of integration for the iterated integral.)

(5 points)

5b) Use your favorite of polar, spherical, or cylindrical coordinates to compute the volume of the R-ball above. (You know you should get $\frac{4}{3}\pi R^3$ for the answer!)

(10 points)