Name Solutions

Student I.D. ______________________

Math 2210-3
Exam #1
February 17, 2010

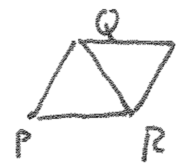
Please show all work for full credit. This exam is closed book, closed note, but you may use a scientific (non-graphing) calculator. There are 100 points possible, as indicated below and in the exam. Since you only have an hour you should be careful to not spend too long on any one problem. Good Luck!!

Score

		POSSIBLE
1	______	20
2	______	20
3	______	25
4	______	35
TOTAL	______	100

1) Consider the three points P=(3,0,0), Q=(2,0,2), and R=(0,-1,4).

1a) Find the area of the triangle with vertices P,Q,R. (10 points)



$\vec{PQ} = \vec{Q} - \vec{P} = \langle -1, 0, 2 \rangle$

$\vec{PR} = \vec{R} - \vec{P} = \langle -3, -1, 4 \rangle$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & 2 \\ -3 & -1 & 4 \end{vmatrix} = \langle 2, -2, 1 \rangle$$

$$\triangle \text{ area} = \tfrac{1}{2} \,\square\, \text{area} = \tfrac{1}{2} \| \vec{PQ} \times \vec{PR} \|$$
$$= \tfrac{1}{2}\sqrt{4+4+1} = \boxed{3/2}$$

1b) Find the equation of the plane containing the points P,Q, and R. (5 points)

$\vec{PQ} \times \vec{PR}$ is normal vector to plane so plane eqtn is

$$2x - 2y + z = 2x_0 - 2y_0 + z_0$$
$$\boxed{2x - 2y + z = 6} \quad \text{(e.g. use P)}$$

1c) The plane whose equation you just found intersects the x-z coordinate plane in a line. Express this line of intersection parametrically. (5 points)

set $y = 0$, yields $2x + z = 6$

$(3,0,0)$ (P)

$(0,0,6)$ (S)

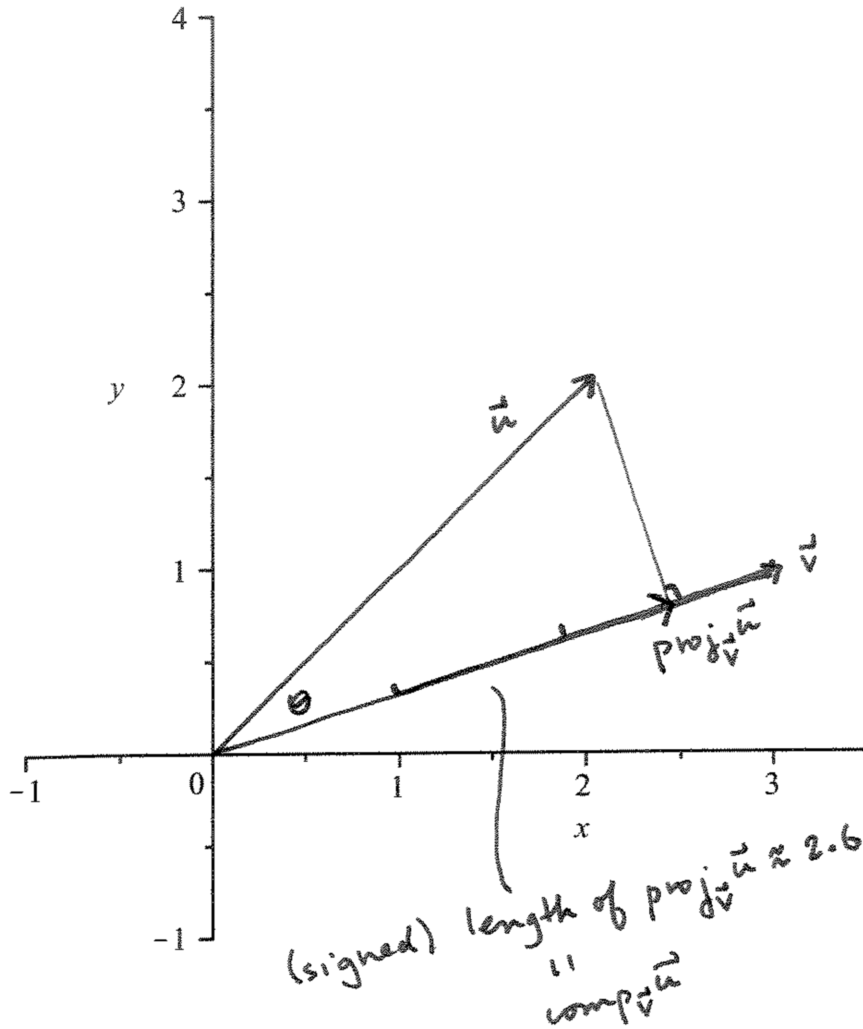
$\vec{F}(t) = P + t\,\vec{PS}$ works

$$\boxed{\vec{F}(t) = \langle 3,0,0 \rangle + t \langle -3, 0, 6 \rangle}$$

2) Let $\mathbf{u} = \langle 2, 2\rangle$ and $\mathbf{v} = \langle 3, 1\rangle$ be two vectors in the plane.

2a) Carefully sketch **u**, **v** as position vectors, onto the coordinate plane below. Use your index card ruler to geometrically add the projection $proj_{\mathbf{v}}\mathbf{u}$ to your picture, and to measure the value of $comp_{\mathbf{v}}\mathbf{u}$.

(12 points)



2b) Use dot products to algebraically compute $proj_{\mathbf{v}}\mathbf{u}$ and $comp_{\mathbf{v}}\mathbf{u}$. Of course, your answers should be consistent with your geometric work in (2a).

(8 points)

$$proj_{\vec{v}}\vec{u} = \left(\vec{u}\cdot\frac{\vec{v}}{\|\vec{v}\|}\right)\frac{\vec{v}}{\|\vec{v}\|} = \frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\vec{v} = \frac{8}{10}\langle 3,1\rangle$$

$$\boxed{proj_{\vec{v}}\vec{u} = \langle 2.4, .8\rangle}$$

$$comp_{\vec{v}}\vec{u} = +\|proj_{\vec{v}}\vec{u}\| = \frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|} = \frac{8}{\sqrt{10}}$$

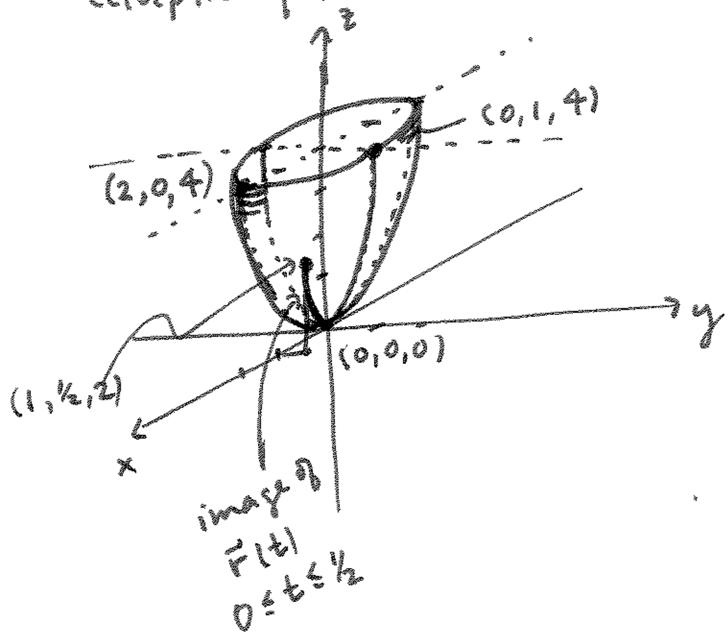
$$\boxed{comp_{\vec{v}}\vec{u} = \frac{8}{\sqrt{10}}}$$

$$\approx \frac{8}{3.2} \approx 2.5$$

3a) Carefully sketch the surface with equation $z = x^2 + 4y^2$. Include accurate trace curves of this surface in the coordinate planes and in the plane $z = 4$.

(10 points)

(elliptic) paraboloid



xz plane : $z = x^2$ CU parabola

yz plane $z = 4y^2$ CU parabola

$z = 0$ (xyplane) point $(0,0,0)$

$z = 4$: $4 = x^2 + 4y^2$ ellipse

$1 = \frac{x^2}{4} + y^2$

3b) Show algebraically that the curve with position vector $\mathbf{r}(t) = \langle 2t, t, 8t^2 \rangle$ lies on the surface of part (3a).

x y z

$x^2 + 4y^2 = 4t^2 + 4t^2 = 8t^2 = z$ ✓

(5 points)

3c) Add the arc of the curve with position vector $\mathbf{r}(t)$ from (3b), to the sketch in (3a), for $0 \le t \le \frac{1}{2}$. Label both endpoints.

$\vec{r}(0) = \langle 0,0,0 \rangle$

$\vec{r}(\tfrac{1}{2}) = \langle 1, \tfrac{1}{2}, 2 \rangle$

(5 points)

3d) Exhibit a definite integral for the length of the arc in (3c). Do not attempt to compute the value of this integral.

(5 points)

$\vec{r}'(t) = \langle 2, 1, 16t \rangle$

$$L = \int_0^{1/2} \|\vec{r}'(t)\| dt = \int_0^{1/2} (4 + 1 + 256t^2)^{1/2} dt = \boxed{\int_0^{1/2} \sqrt{5 + 256t^2}\, dt}$$

4) Consider the particle motion parametric curve with position vector $\mathbf{r}(t) = \langle 2e^{-t}, e^{2t} \rangle$, at time t.

4a) Compute the position, velocity, acceleration, and speed, when $t = 0$.

(10 points)

$\vec{r}\,'(t) = \langle -2e^{-t}, 2e^{2t} \rangle$

$\vec{r}\,''(t) = \langle 2e^{-t}, 4e^{2t} \rangle$

$v(t) = \|\vec{r}\,'(t)\| = 2\sqrt{e^{-2t} + e^{4t}}$

$$\boxed{\begin{array}{l} \vec{r}(0) = \langle 2, 1 \rangle \\ \vec{r}\,'(0) = \vec{v}(0) = \langle -2, 2 \rangle \\ \vec{r}\,''(0) = \vec{a}(0) = \langle 2, 4 \rangle \\ v(0) = 2\sqrt{2} \end{array}}$$

4b) Use your favorite formula to compute the curvature at the point on the curve where $t = 0$.

(5 points)

for plane curve,

$\kappa = \dfrac{|x'y'' - x''y'|}{v^3}$ @ $t=0$ $\kappa = \dfrac{|-2\cdot 4 - 2\cdot 2|}{(2\sqrt{2})^3}$

$= \dfrac{12}{16\sqrt{2}} = \boxed{\dfrac{3}{4\sqrt{2}}} \approx .54$

$\sqrt{2} \approx 1.4$

$4\sqrt{2} \approx 5.6$

$$\begin{array}{r} .5 \\ 5.6\overline{)\,3.0.0} \\ \underline{2\,8\,0} \\ 200 \end{array}$$

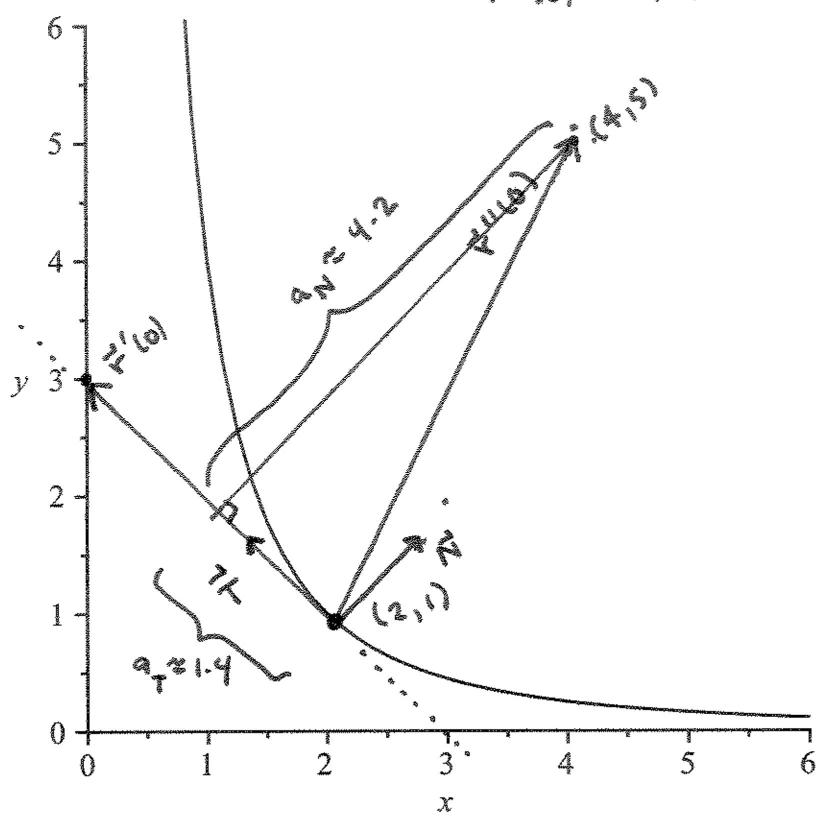
4c) Use a paper ruler to carefully plot the point, velocity vector and acceleration vector for the curve, when $t = 0$. Compute the unit tangent and normal vectors, T and N, at this instant, and add them to the sketch as well.

(10 points)

$\vec{r}(0) = \langle 2, 1\rangle$
$\vec{r}\,'(0) = \langle -2, 2\rangle$
$\vec{r}\,''(0) = \langle 2, 4\rangle$

$\vec{T}(0) = \dfrac{\vec{r}\,'(0)}{\|\vec{r}\,'(0)\|} = \frac{1}{\sqrt{2}}\langle -1, 1\rangle$

$\vec{N}(0) \perp \vec{T}(0)$ & points in dir. of bending.
so $\vec{N}(0) = \frac{1}{\sqrt{2}}\langle 1, 1\rangle$



4d) Geometrically measure and the tangential and normal components of acceleration when $t = 0$, using your sketch above and a suitable right triangle.

(4 points)

$a_T \approx 1.4$
$a_N \approx 4.2$

4e) Compute the tangential and normal components of acceleration algebraically (at $t = 0$), using your favorite method. Your answer should be consistent with your measurements in (4d)!

(6 points)

$a_T = \vec{a}\cdot\vec{T} = \langle 2, 4\rangle \cdot \frac{1}{\sqrt{2}}\langle -1, 1\rangle = \frac{2}{\sqrt{2}} = \sqrt{2} \approx 1.4$ ✓
$a_N = \vec{a}\cdot\vec{N} = \langle 2, 4\rangle \cdot \frac{1}{\sqrt{2}}\langle 1, 1\rangle = \frac{6}{\sqrt{2}} = 3\sqrt{2} \approx 4.2$

(alternately, from RCE, $a_T = v'(0)$, $a_N = \kappa v^2$)