Practice Exam Solutions, First Exam

2210-4 2/14/05

1. $P=(1,2,-1)$ $\quad \vec{PQ} = \langle -1,1,1\rangle$

$Q=(0,3,0)$

$R=(-2,5,1)$

1a) so $\langle -1,1,1\rangle$ is direction vector for line

$\vec{r}(t) = \langle 1,2,-1\rangle + t\langle -1,1,1\rangle$ is position vector

also o.k.: $x=1-t$
$y=2+t$
$z=-1+t$

1b) $A = \frac{1}{2}\,|\vec{PQ}\times\vec{PR}|$.

$$\vec{PQ}\times\vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 1 \\ -3 & 3 & 2 \end{vmatrix} = \langle -1,-1,0\rangle$$

so $\boxed{A = \frac{1}{2}\sqrt{2} = \frac{1}{\sqrt{2}}}$

1b) See above; plane has normal vector $\langle -1,-1,0\rangle$, hence also $\langle 1,1,0\rangle$

Eqn $\quad 1\cdot(x-1) + 1\cdot(y-2) + 0(z+1) = 0$

(using P)

or $\boxed{x+y=3}$

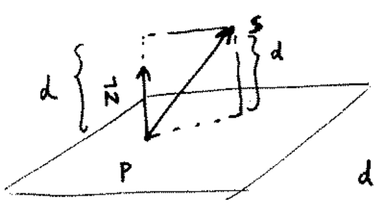
1d) dist $= \dfrac{|\vec{PS}\cdot\vec{N}|}{|\vec{N}|}$ $\qquad \vec{N} = \langle 1,1,0\rangle$

$S=(1,2,3)$
$P=(1,2,-1)$
$\vec{PS} = \langle 0,0,4\rangle$

$\vec{PS}\cdot\vec{N} = \langle 0,0,4\rangle\cdot\langle 1,1,0\rangle = 0$, $\boxed{\text{so dist} = 0}$

so S is on the plane!

Oh, yeah it satisfies $x+y=3$

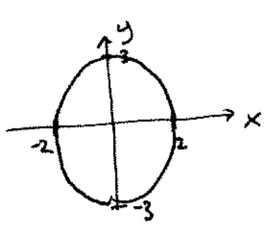
1e) 

$$d = |\text{comp}_{\vec{N}}\vec{PS}| = \left|\vec{PS}\cdot\frac{\vec{N}}{|\vec{N}|}\right| = \frac{|\vec{PS}\cdot\vec{N}|}{|\vec{N}|}$$

2 a) $\dfrac{x^2}{4} + \dfrac{y^2}{9} - z^2 = 1$

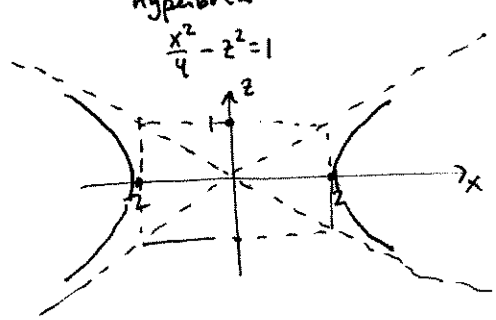
trace curves

xy plane
($z=0$)

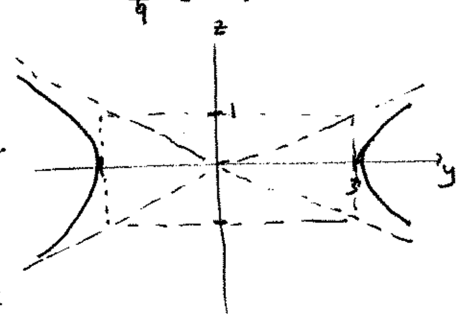


ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$

xz plane
($y=0$).
hyperbola
$\frac{x^2}{4}-z^2=1$

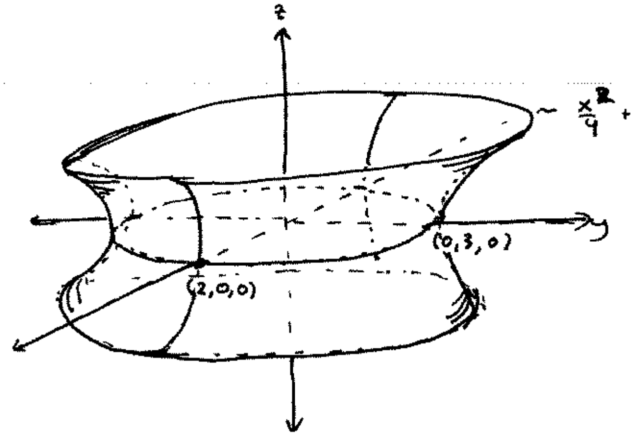


yz plane
$x=0$
$\frac{y^2}{9}-z^2=1$



2b) 1-sheeted hyperboloid – can deduce this from the trace curves

2c)



3 $\vec{r}(t)=\langle e^t, e^{-2t}\rangle$

3a) $x=e^t$

$x^2=e^{2t}$

$\frac{1}{x^2}=e^{-2t}=y$

so all points on curve satisfy $y=\frac{1}{x^2}$

3b) $\vec{r}\,'(t)=\langle e^t, -2e^{-2t}\rangle$

$\vec{r}\,''(t)=\langle e^t, 4e^{-2t}\rangle$

$v(t)=|\vec{r}\,'(t)|=\sqrt{e^{2t}+4e^{-4t}}$

3c) $\vec{r}\,'(0)=\langle 1,-2\rangle$ $\quad v(0)=\sqrt{5}$

$\vec{r}\,''(0)=\langle 1,4\rangle$

$\kappa=\frac{|x'y''-x''y'|}{v^3}=\frac{|1\cdot 4-(-2)\cdot 1|}{5^{3/2}}=\frac{6}{5^{3/2}}$

3d) $\vec{r}(0)=\langle 1,1\rangle$

$\vec{r}\,'(0)=\langle 1,-2\rangle$

$\vec{r}\,''(0)=\langle 1,4\rangle$

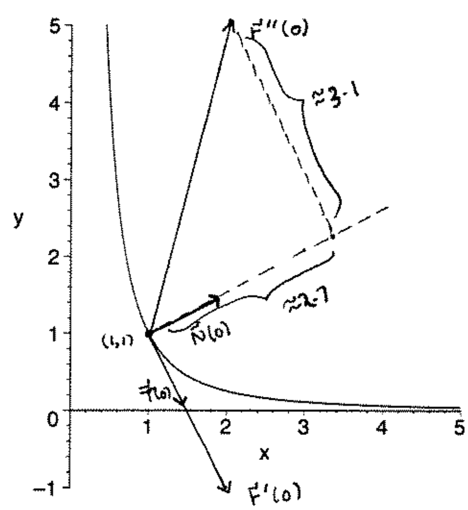
$\vec{T}(0)=\frac{\vec{r}\,'(0)}{|\vec{r}\,'(0)|}=\frac{1}{\sqrt{5}}\langle 1,-2\rangle$

$\vec{N}=\langle 2,1\rangle\frac{1}{\sqrt{5}}$

3e) $\text{comp}_{\vec{T}}\vec{r}\,''(0)\approx -3.1$

(because go about 3.1 units in $-\vec{T}$ direction

$\text{comp}_{\vec{N}}\vec{r}\,''(0)\approx 2.7$



3f) $\text{comp}_{\vec{T}}\vec{r}\,''(0)=\vec{r}\,''(0)\cdot\vec{T}$

$=\langle 1,4\rangle\cdot\frac{1}{\sqrt{5}}\langle 1,-2\rangle$

$=\frac{1}{\sqrt{5}}(1-8)=\boxed{-7/\sqrt{5}}\approx -3.13$

$\text{comp}_{\vec{N}}\vec{r}\,''(0)=\vec{r}\,''(0)\cdot\vec{N}$

$=\langle 1,4\rangle\cdot\frac{1}{\sqrt{5}}\langle 2,1\rangle$

$\boxed{=\frac{1}{\sqrt{5}}6}\approx 2.68$

3g) $\vec{r}\,''(t)=v'(t)\vec{T}+\kappa v^2\vec{N}$

$v(t)=(e^{2t}+4e^{-4t})^{1/2}$

$v'(t)=\frac{1}{2}(e^{2t}+4e^{-4t})^{-1/2}(2e^{2t}-16e^{-4t})$

$v'(0)=\frac{1}{2}5^{-1/2}(-14)=\boxed{-7/\sqrt{5}}$ ✓

$\kappa(0)=6/5^{3/2}$

$v(0)=5^{1/2}$

$\kappa v^2=\frac{6}{5^{3/2}}\cdot 5=\boxed{6/\sqrt{5}}$ ✓