

Name.....
I.D. number.....

Math 2210-4

Practice Exam 1

February 14, 2005

The exam will be closed-book and closed-note. You will be allowed the use of a scientific calculator (only), but not one which is capable of graphing or of solving linear algebra equations. You won't actually need the calculator, but there could be times when numerical values would be a hint about whether you've correctly solved a problem. **In order to receive full or partial credit on any problem, you must show all of your work and justify your conclusions.** There are 100 points possible. The point values for each problem are indicated in the right-hand margin. On the actual test there will be space for your work. **Good Luck!**

1) Consider the three points $P = (1, 2, -1)$, $Q = (0, 3, 0)$ and $R = (-2, 5, 1)$.

1a) Find a parametric formula for the line through P and Q .

(5 points)

1b) Find an equation for the plane containing P, Q and R .

(15 points)

1c) Find the area of the triangle having P, Q, R as vertices.

(5 points)

1d) Find the distance from the plane in (1a) to the point $S = (1, 2, 3)$. You may use the formula

$$dist = \frac{|PS \cdot N|}{|N|}$$

where P is any point on the plane, and N is a perpendicular vector to the plane.

(5 points)

1e) Explain, using your understanding of the dot product and an appropriate picture, why the formula for distance in (1c) is true.

(5 points)

2) Consider the surface which is the graph of the equation

$$\frac{x^2}{4} + \frac{y^2}{9} - z^2 = 1$$

2a) Sketch the trace curves of this surface in the three coordinate planes

(10 points)

2b) Identify the type of this quadric surface

(5 points)

2c) Sketch the surface.

(5 points)

3) Consider the parametric curve with position vector given by

$$\mathbf{r}(t) = \langle \mathbf{e}^t, \mathbf{e}^{(-2t)} \rangle.$$

There is a sketch of part of the curve below.

3a) Do a short computation to show that this curve lies on the graph $y = \frac{1}{x^2}$, in the x - y plane.

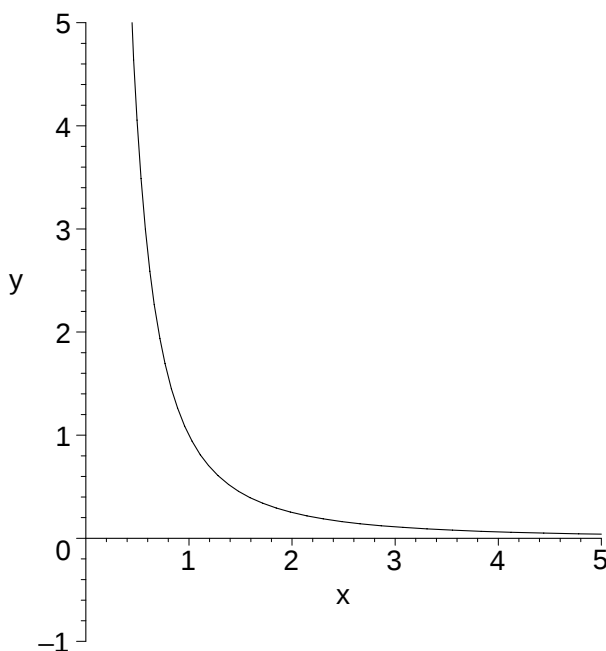
(5 points)

3b) Compute $\mathbf{r}'(t)$, $\mathbf{r}''(t)$, and the speed $v(t)$.

(5 points)

3c) Using your favorite formula, find the curvature of this curve at the point where $t=0$.

(5 points)



3d) Label the point with position vector $\mathbf{r}(0)$ into the picture above. Compute, and accurately draw the vectors $\mathbf{r}'(0)$ and $\mathbf{r}''(0)$ into the picture, in the appropriate location(s), using an index card ruler. Also, compute and draw in the unit tangent and normal vectors, T and N , when $t=0$. (Hint: you can find N easily because it is perpendicular to T and the curve lies in a plane.)

(10 points)

3e) Use your sketch, an appropriate right triangle which you add to the picture, and an index card ruler to measure and record (accuracy within 0.2 suffices) the components of the acceleration $\mathbf{r}''(0)$ in the tangential and normal directions.

(5 points)

3f) Use the dot product to find the exact values of the acceleration components in the tangential and normal directions. (These should be close to your numerical approximations in (3e)!

(8 points)

3g) Refind the components in (3f) by using the roller coaster equation

$$\mathbf{r}''(t) = \left(\frac{d}{dt} v(t) \right) T + \kappa v^2 N$$

where the scalar v represents speed and κ is the curvature. Note that you've already done most of the computations you need in (3b), (3c).

(7 points)