

Postpone the last 2 problems in §15.8; # 28, 29, until next assignment.

§15.8

Max-min problems  
for fns of several variables

Notation

Let  $f: D \rightarrow \mathbb{R}$

$f(x, y)$   
 $f(x, y, z)$  etc.

$f(\vec{p}_0)$  is global max. value  
if  $f(\vec{p}_0) \geq f(p) \forall p \in D$

global min. value  
if  $f(\vec{p}_0) \leq f(p) \forall p \in D$

$f(\vec{p}_0)$  is a local max or min value  
if there is a neighborhood  
 $N$  of  $\vec{p}_0$  so that

$f(\vec{p}_0)$  is the max or  
min value of  $f$  is  $D \cap N$

Theorem: If  $D$   
is a closed and bounded domain

↑  
includes  
its boundary

↑  
all points are  
within some fixed  
finite distance of  
origin

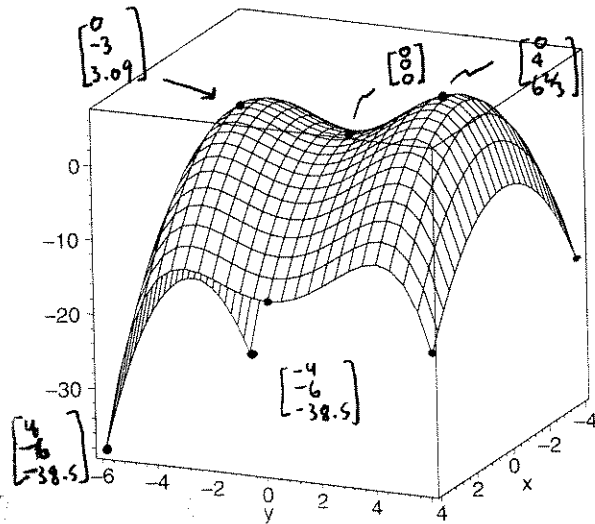
And if  $f$  is continuous at all points of  $D$   
Then  $f$  attains its global maximum  
and minimum values. This either  
happens at boundary points or  
interior points. If the extreme  
value is  $f$  of an interior point  $p_0$   
then either

(a)  $\nabla f(p_0) = 0$

or (b)  $f$  is not diffble at  $p_0$

$z = f(x, y)$

$z = \frac{3}{4}y^2 + \frac{1}{24}y^3 - \frac{1}{32}y^4 - x^2$



Domain  $-4 \leq x \leq 4$   
 $-6 \leq y \leq 4$

i.e. a closed rectangular  
domain  
↑  
contains its  
boundary

Example 1 By inspection above

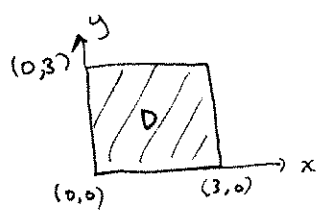
① Find all global extrema

② Find some local max & min values  
which are not global extrema

③ What's interesting about  $(0, 0, 0)$ ?

Example 2

$$D = \{(x,y) \mid 0 \leq x \leq 3, 0 \leq y \leq 3\}$$

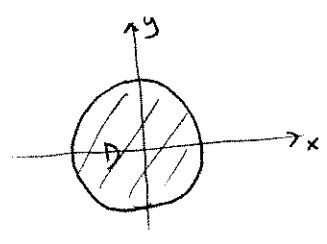


Find the global max and min values of  $f(x,y) = 3x^2 - 6x + 2y^2 - 8y + 13$

Example 3

$$D = \{(x,y) \mid x^2 + y^2 \leq 1\}$$

Find the global max and min values of  $f(x,y) = x^2 + y^2 + 3xy$



second derivative test, for  $f(x,y)$ :

If  $\nabla f(p) = 0$

Compute the Hessian matrix  $\begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix}$

and let  $D = f_{xx}f_{yy} - f_{xy}^2$  be its determinant at  $p$

If

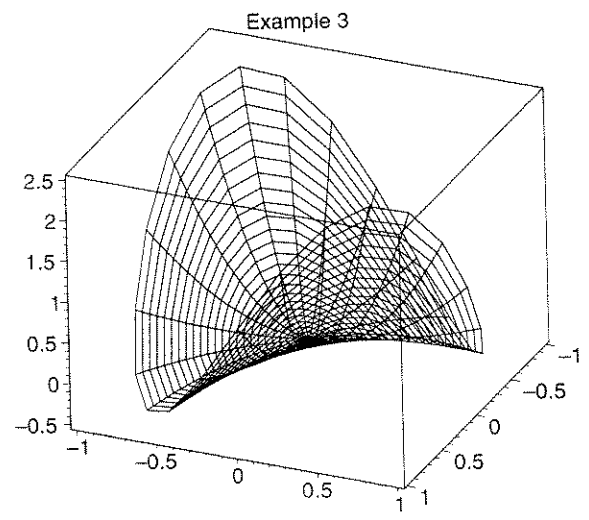
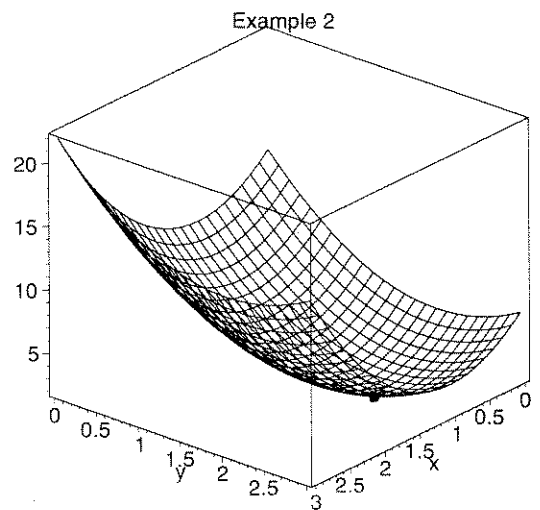
(1)  $f_{xx} > 0$  AND  $D > 0$   $f$  is concave up  $\rightarrow$   $f(p)$  local min

(2)  $f_{xx} < 0$  AND  $D > 0$   $f$  is CD  $\rightarrow$   $f(p)$  local max

(3)  $D < 0$   $f(p)$  is a saddle point neither local max or min

(4)  $D = 0$  can't say anything w/o more work

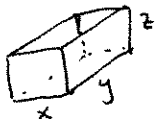
Check Examples 2 & 3 critical pts with 2nd deriv test:



# Applications (more in HW!)

4

## Example 4



open-topped box made of cardboard

Volume to be  $4 \text{ m}^3$

What choices of  $x, y, z$  minimize required surface area of cardboard?

Example 5 Find the nearest point on the paraboloid  $z = x^2 + y^2$ , to  $P = (2, 0, 0)$

