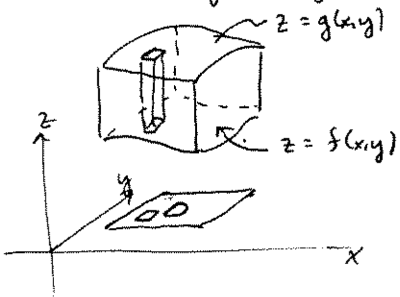
Math 2210-4
Wed 30 March

Review session for exam: Friday 4/1, 2:30–3:30 pm, JWB 335

I'll email you (& post) a practice exam later tonight.

§ 16.5 applications of double integrals

1 Volume of a region between 2 graphs



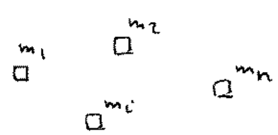
$$dV = (g(x,y) - f(x,y))\,dA$$

$$V = \iint_D g(x,y) - f(x,y)\,dA$$

Example: you've done lots of examples where the lower graph was just $z = 0$.

2 Mass of lamina

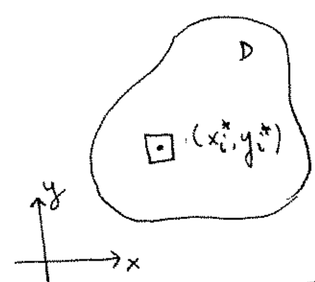
Discrete.



total mass

$$m = \sum_{i=1}^{n} m_i$$

lamina



$\delta(x,y)$ = density fcn

units mass/area

$$m \approx \sum_i \underbrace{\delta(x_i^*, y_i^*)}_{(\text{mass}/\text{area})} \underbrace{\Delta A_i}_{(\text{area})}$$

Riemann sum

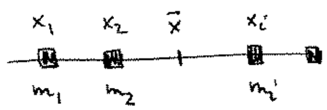
$$\boxed{m = \iint_D \delta(x,y)\,dA}$$

Discrete

) centers of mass

on a line (teeter totter)

$\bar{x}$ satisfies the no net torque condition



t
que
t $\bar{x}$:

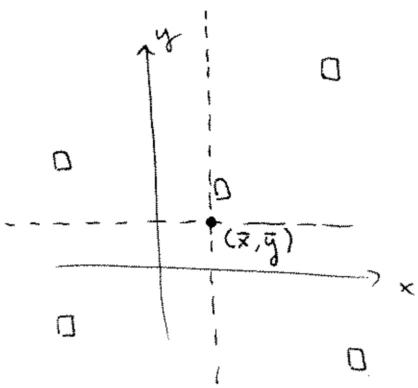
$$\sum_i m_i (x_i - \bar{x}) = 0$$

i.e.

$$\sum_i m_i x_i - \bar{x} \sum m_i = 0$$

$$\boxed{\bar{x} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\sum m_i x_i}{m}}$$

masses in a plane have center of mass $(\bar{x}, \bar{y})$ (balance point)



$$\sum m_i (x_i - \bar{x}) = 0$$

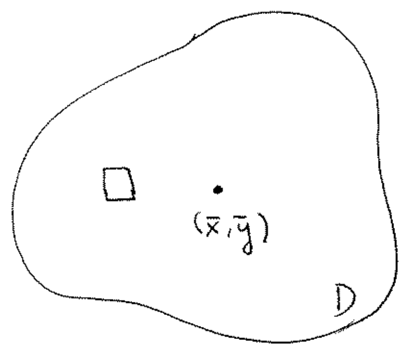
$$\& \sum_i m_i (y_i - \bar{y}) = 0$$

So
$$\boxed{\begin{aligned} \bar{x} &= \frac{\sum m_i x_i}{m} = \frac{M_y}{m} \\ \bar{y} &= \frac{\sum m_i y_i}{m} = \frac{M_x}{m} \end{aligned}}$$

moment wrt y-axis!

moment wrt x-axis

Lamina



$(\bar{x}, \bar{y})$ defined by

$$\iint_D (x - \bar{x})\, \delta(x,y)\, dA = 0$$

$$\iint_D (y - \bar{y})\, \delta(x,y)\, dA = 0$$

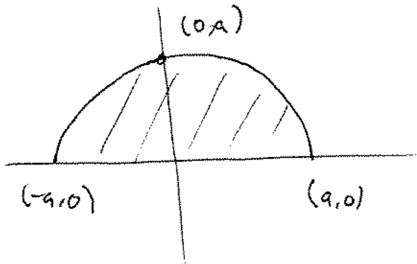
yields

$$\boxed{\begin{aligned} \bar{x} &= \frac{\iint_D x\, \delta(x,y)\, dA}{m} \\ \bar{y} &= \frac{\iint_D y\, \delta(x,y)\, dA}{m} \end{aligned}}$$

$\left(= \frac{M_y}{m}\right)$

$\left(= \frac{M_x}{m}\right)$

<u>Example</u>: Consider a half disk with constant density δ : $x^2+y^2 \leq a^2$
$y \geq 0$



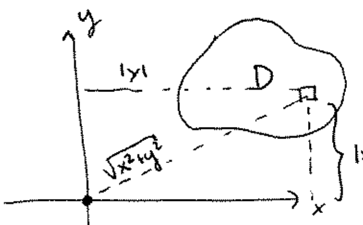
- explain why $\bar{x} = 0$

- find $\bar{y}$

ans $\bar{y} = \frac{4}{3\pi} a \approx .424 a$

4 Moments of intertia I_x, I_y, I_z

(Used to calculate kinetic energy when object is rotated about an axis; $KE = \frac{1}{2} I \omega^2$



(z-axis pointing at you)

$$I_x = \iint_D y^2 \, \delta(x,y)\, dA \qquad \left(\text{Note } KE = \iint_D \tfrac{1}{2}(y\omega)^2 \delta(x,y)\, dA = I_x \omega^2\right)$$

$$I_y = \iint_D x^2 \, \delta(x,y)\, dA$$

$$I_z = \iint_D (x^2+y^2)\, \delta(x,y)\, dA \qquad (= I_x + I_y\,!)$$

<u>example</u>: Find I_x for the half disk in page 3 example

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> assume(a,positive):
  simplify(int(int(y^2,y=0..sqrt(a^2-x^2)),x=-a..a),radical);
```

Int table #62

$$\frac{a\sim^4 \pi}{8}$$

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> int(int(r^3*sin(theta)^2,theta=0..Pi),r=0..a);
```

Int table #20

$$\frac{a\sim^4 \pi}{8}$$

Review Sheet for second exam

Math 2210-4, March 30, 2005

Our second exam is on Monday April 4, in class, from 11:50-12:50. The exam will cover sections 14.7-16.5 from our text. Of course we are building on the material from earlier in the course, so you'll be expected to be comfortable with vectors, curves, chain rule, etc. In the outline below you are responsible for each topic! We will go through this sheet on Friday April 1.

Cylindrical, **Spherical** (and **polar**) coordinates. (14.7)
- converting between these coordinate systems and Cartesian (x,y,z) coordinates
- describing surfaces and regions using these coordinate systems

Differentiability for functions of several variables (Chapter 15)
- functions of two or three variables (15.1)
  - the graph z=f(x,y). Contours on the graph.
  - level curves of f(x,y), in the domain
  - level surfaces of the function f(x,y,z), in the domain
- partial derivatives (15.2)
  - definition:
  - meaning in terms of rate of change of function in coordinate directions.
  - how to compute partial derivatives
  - interpretation of partial derivatives as slope of trace curve, for functions of two variables.
  - higher order partial derivatives; equality of mixed partials.
- limits and continuity (15.3)
  - definitions, meaning.
- differentiability (15.4-15.7)
  - definition and meaning:
  - which functions are differentiable?
  - tangent function, tangent plane
  - differential approximation (15.7)
- directional derivatives (15.5)
  - definition and meaning
  - how to compute with the gradient
  - relation of gradient to level curves and level surfaces - tangent planes revisited
- chain rule! (15.6)
- Max-min problems (15.8-15.9)
  - Continuous functions on closed and bounded sets attain their extrema - where?
  - Critical points in the interior - how to find them?
  - Second derivative test
  - Lagrange method for constrained optimization problems (15.9)

Multiple Integration (Chapter 16)
- Double integrals over rectangular domains
  - definition and meaning, properties (16.1)
  - how to compute as iterated integrals (16.2)
- Double integrals over non-rectangular domains (16.3-16.4)
  - iterated integrals for vertically or horizontally simple domains (16.3)
  - double integrals in polar coordianates (16.4)
- Applications of double integrals (16.3, 16.5)
  - volumes, mass, center of mass, moments of inertia