Math 2210-1
Fri Mar 11.

HW for Fri 3/25
15.8 28, 29
15.9 1, 3, 5, 9 10, 11 Do 9 both ways: Lagrange & the §15.8 way.

①

$\boxed{\text{More to come!}}$

§15.9 Lagrange's method
Another way to do constrained max-min problems (in 1 or more variables)

Remember our (my) favorite Calculus problem:

Find the rectangle of area one which has minimum perimeter

minimize $P(x,y) = 2x + 2y$

subject to $xy = 1 = A(x,y)$

old way: $y = \frac{1}{x}$

minimize $f(x) = 2x + \frac{2}{x}$

critical pts: $f'(x) = 0 = 2 - \frac{2}{x^2}$

$2 = \frac{2}{x^2}$

$x^2 = 1$

$x = 1 \Rightarrow y = 1$

square!

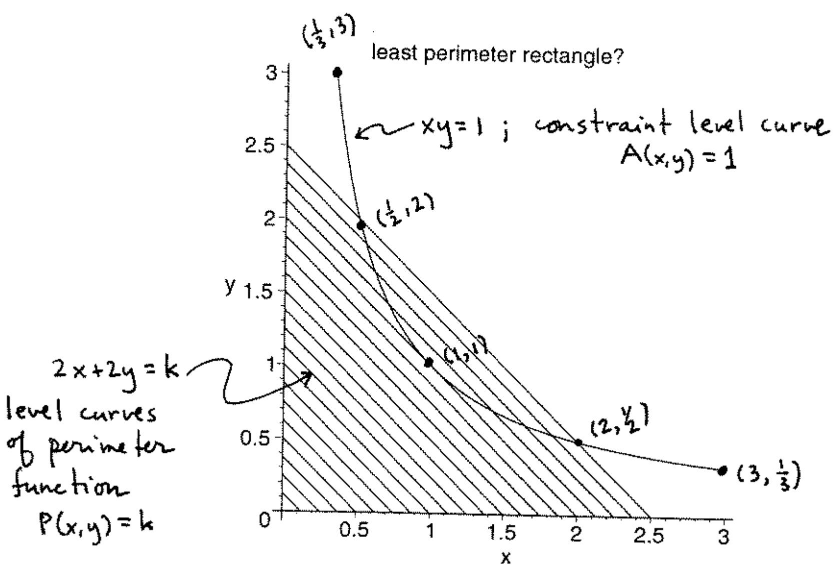
New way (Lagrange's method)

What do you notice about

∇P and ∇A

and the point $(1,1)$?

Explain why this must be so!



Lagrange multipliers

If $F(p_0)$ is a maximum or minimum value of $F(x,y)$ on the level curve $G(x,y)=k$
$F(x,y,z)$ level surface $G(x,y,z)=k$
etc. etc.

Then $\nabla F(p_0) = \lambda \nabla G(p_0)$

for some scalar λ, called the Lagrange multiplier.

example Minimize $P(x,y) = 2x+2y$
subject to $A(x,y) = xy = 1$
using Lagrange multipliers.

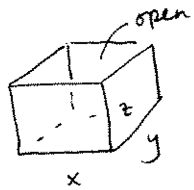
example (from §15.8 #13)(hw due today)
original problem was to find max & min values
of $f(x,y) = x^2-y^2+1$ in the disk $\{(x,y) \text{ s.t. } x^2+y^2 \le 1\}$.
the only critical point was $(0,0)$, but it's a saddle.
So max and min occur on the circle $g(x,y) = x^2+y^2 = 1$.
Now, use Lagrange!

Why Lagrange multipliers works:
Let $F(p_0)$ be a max or min value of $F(p)$ on the level surface $G(p) = \text{const}$.
Let $\vec{r}(t)$ be any curve (with range) on the level surface $G(p) = \text{const}$, i.e. $G(\vec{r}(t)) = \text{const}$
If $\vec{r}(t_0) = \vec{p}_0$ and $F(\vec{p}_0)$ is a max or min value
then $\vec{F}(\vec{r}(t_0))$ is a local max or min for $\vec{F}(\vec{r}(t))$

$\Rightarrow (F \circ \vec{r})'(t_0) = 0 = \nabla F(\vec{p}_0) \cdot \vec{r}\,'(t_0)$

Deduce $\nabla F(\vec{p}_0) \perp$ level $\left(\begin{smallmatrix}\text{curve or}\\ \text{surface}\end{smallmatrix}\right)$. But so is $\nabla G(\vec{p}_0)$. $\Rightarrow \nabla F(\vec{p}_0) = \lambda \nabla G(\vec{p}_0)$ ∎

Example Rework §15.8 # 19 With Lagrange.



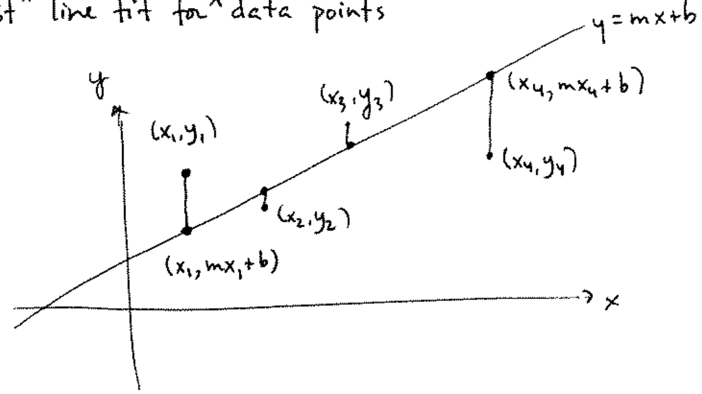
$V = xyz = 256 \quad ft^3$

$SA = xy + 2xz + 2yz$ to be minimized

in HW due today you obtained $x = y = 8;\ z = 4.$

Linear regression : this is an unconstrained max-min problem we didn't have time for on Wed.

"best" line fit for n data points



$f(m,b)$ = sum of squared vertical deviations

$$= \sum_{i=1}^{n} (y_i - mx_i - b)^2$$

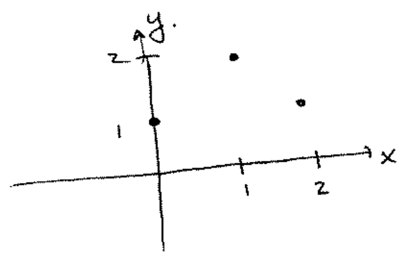
Find m, b to minimize $f(m,b)$. Then $y = mx+b$ is called the linear regression line.

In HW 28 you show the critical pt (m_0, b_0) satisfies

$$\boxed{\begin{aligned} m\left(\textstyle\sum x_i^2\right) + b\left(\textstyle\sum x_i\right) &= \textstyle\sum x_i y_i \\ m \textstyle\sum x_i \quad + nb \quad &= \textstyle\sum y_i \end{aligned}}$$

↑ book typo there! Says $\sum x_i y_i$

Example : Find best line fit for $\{(0,1), (1,2), (2,1)\}$



$n = 3$

$\sum x_i =$

$\sum y_i =$

$\sum x_i y_i =$

$\sum x_i^2 =$