Math 2210-4
Monday 1/31

Homework:

Due Friday, February 4:
14.6 2, 5, 7, 8, 9, 13 14, 21, 22, 25. Note several additional problems from last week.
13.1 3, 6, 9, 13, 17. In 3, 6 also verify that dy/dx = (dy/dt)/(dx/dt) by computing both expressions directly.
14.4 23, 25.
14.5 1, 7, 15, 23. In 15, 23 also sketch the curves. For 15 sketch the curve onto the cylinder y^2+z^2=1. For 23 use the auxillary "cylinder" y^2-x^2=1, whose cross section is a hyperbola.

We discussed parametric lines, with position vector (from the origin to the point)

$\vec{r}(t) = r_0 + t\vec{v}$

$\vec{r}(t) = \langle x_0, y_0, z_0\rangle + t\langle a, b, c\rangle$

the position vector notation is a more compact and useful description of the curve than the equivalent notation

$x = x_0 + at$
$y = y_0 + bt$
$z = z_0 + ct$

Today we begin discussing general parametric curves (§13.1, 14.4, 14.5)

$\vec{r}(t) = \langle f(t), g(t), h(t)\rangle$
$= f(t)\hat{i} + g(t)\hat{j} + h(t)\hat{k}$

position vector description

$x = f(t)$
$y = g(t)$
$z = h(t)$

equivalent description

- often an interval is specified for the t values, i.e. the domain of $\vec{r}(t)$: $a \le t \le b$

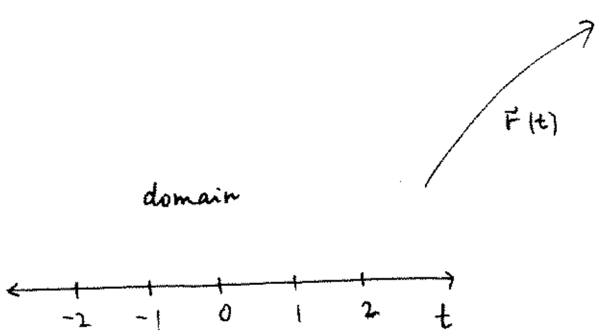
- if the curve lies in the x-y plane $\vec{r}(t) = \langle f(t), g(t)\rangle$ i.e. $x = f(t)$, $y = g(t)$

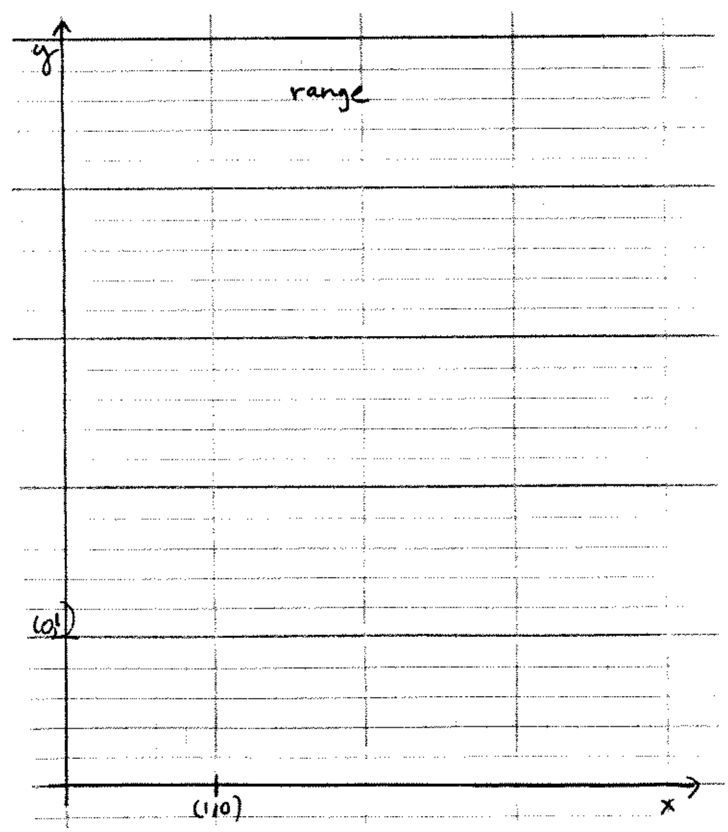
Example 1 $\vec{r}(t) = \langle 2+t, 1+t^2\rangle$
$-2 \le t \le 2$

(a) plot some points on this curve

(b) Find the Cartesian (x-y) eqtn satisfied by all points on the curve, then sketch it ("Solve for t")

t	$\vec{r}(t)$
-2	
-1	
0	
1	
2	





Example 2

If the curve $x = f(t)$, $y = g(t)$ is locally a graph $y = F(x)$, then the slope of this graph is

near some given point

$$F'(x) = \frac{dy}{dx}$$

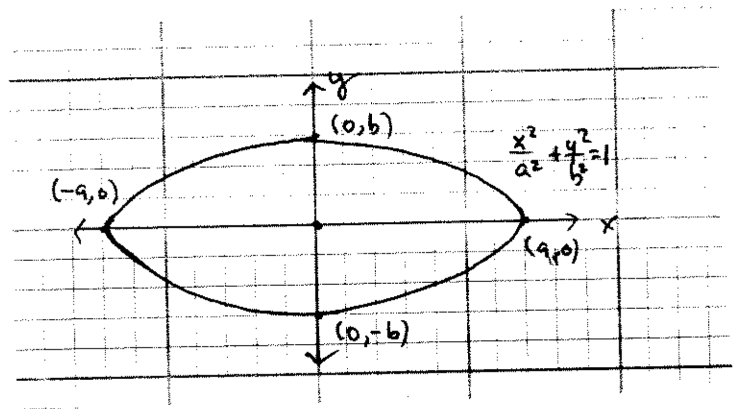
but x, y are also fcns of t, so

$$\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} \qquad \text{1210 chainrule!}$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{dy/dt}{dx/dt}}$$

Compute $\frac{dy}{dx}$ both ways (see box)

for Example 1, at the point $(1,2)$



Some favorite curves in the plane:

Example 3 Show $\boxed{\vec{r}(t) = \langle a\cos t,\ b\sin t\rangle}$ $(a, b > 0)$

is the position vector of a curve traveling around the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

What is a good domain t-interval so that $\vec{r}(t)$ describes a <u>simple</u> ← "$\vec{r}(t)$ doesn't cross itself" i.e. no self intersections, $\vec{r}(t)$ is one to one

<u>closed</u> curve, traveling once around the ellipse?

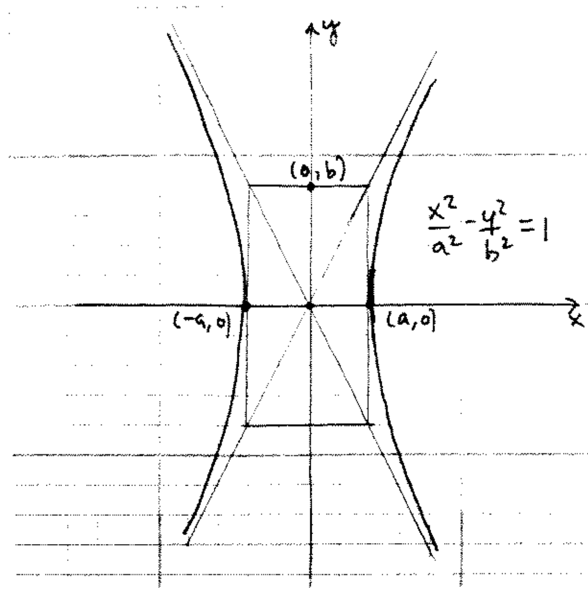
starts & ends at the same place, $\vec{r}(a) = \vec{r}(b)$

Example 4 Show $\vec{r}(t) = \langle a\cosh t, b\sinh t \rangle$ $(a, b > 0)$

parameterizes half of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

How could you parameterize the other half?



Recall,

$\cosh t = \frac{1}{2}(e^t + e^{-t})$

$\sinh t = \frac{1}{2}(e^t - e^{-t})$

$\cosh^2 t - \sinh^2 t = 1$!

Example 5: Sketch the curve of intersection between the cylinder

$x^2 + y^2 = 4$

and the plane $y + z = 2$

Then, find a parametric representation of this curve!