Math 2210-4
Wed 26 Jan.

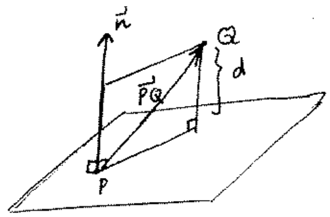
We will use today to discuss lines in $\mathbb{R}^3$ (pages 4-5 Monday notes)
and then to use the dot and cross products to answer natural questions
about line and plane geometry. This will also review essential dot & cross product properties

- pages 4-5 Monday.

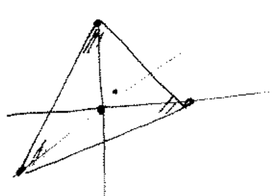
- 28, 29, 30 § 14.4 HW: distances between points, planes, lines

(28) P a point in a plane with normal vector $\vec{n}$
Q a point
d = distance from Q to the plane: Find formula for d using

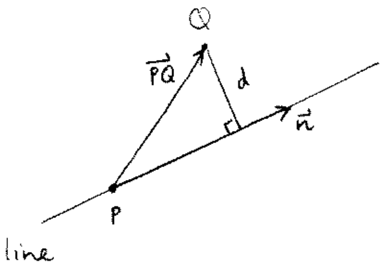
$$\boxed{\begin{aligned} \mathrm{proj}_{\vec{b}}\,\vec{a} &= \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\,\vec{b} \\ \mathrm{comp}_{\vec{b}}\,\vec{a} &= \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} \end{aligned}}$$



Example 1
Find the distance from the plane $x+y+z=3$ to the origin



(29) Point to line, nearest distance : Use relationship between cross product and parallelogram area to deduce the formula



$$d = \frac{|\vec{PQ} \times \vec{n}|}{|\vec{n}|}$$

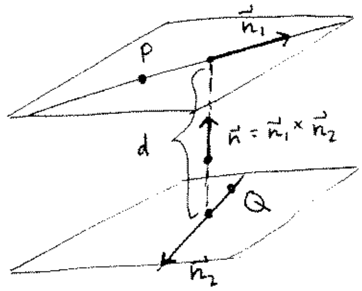
where $\vec{n}$ is a direction vector for the line

Example 2 Find the distance from the line $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-1}{1}$ to the point $Q = (5, 4, 3)$:)

(30) Distance between skew lines

$\uparrow$ non-intersecting, non-parallel.

i.e. direction vectors not scalar multiples



Explain $d = \dfrac{|\vec{PQ} \cdot \vec{n}|}{|\vec{n}|}$ ← abs val. ; ← mag ; $\vec{n} = \vec{n}_1 \times \vec{n}_2$

Example 3 Find the distance between the lines $\vec{r}(t) = \langle 1,-2,0\rangle + t\langle 2,3,-4\rangle$ and $\vec{\rho}(s) = \langle -1,-5,4\rangle + s\langle 1,2,-3\rangle$ ☺ $(\vec{n} = \langle -1,2,1\rangle)$