Math 2210-4
Monday 1/24

- Cross product continued §14.3
- lines and planes §14.4

Recall:
cross product definition:

geometric characterization:

Recall:
We computed
$$\langle 2,1,0\rangle \times \langle 1,3,0\rangle = \langle 0,0,5\rangle$$
using determinant def.

1
Check (alternate computation)
$$(2\hat{i} + \hat{j}) \times (\hat{i} + 3\hat{j})$$
using algebra properties
(This was 4 Fri, which we didn't get to.)

continue with pages 3,4 Friday, including Volumes on page 4.

Example 2

Consider points

$P = (1,0,3)$
$Q = (0,1,1)$
$R = (2,1,0)$

(a) Find the equation of a plane thru these 3 points

(b) Find the area of the triangle having P, Q, R as vertices

Lines in $\mathbb{R}^2$ and in $\mathbb{R}^3$.

In $\mathbb{R}^2$:

you all know slope-intercept $(y = mx + b)$
point-slope $(y - y_0 = m(x - x_0))$
ways of writing lines.
(also $ax + by = c$)

There is another way, which works for lines in any dimension, with an easy modification

parametric lines

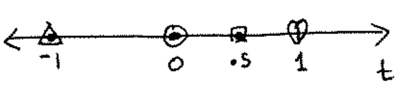
$x = x_0 + at$
$y = y_0 + bt$
(t any real number) i.e. the parameter

Example 2

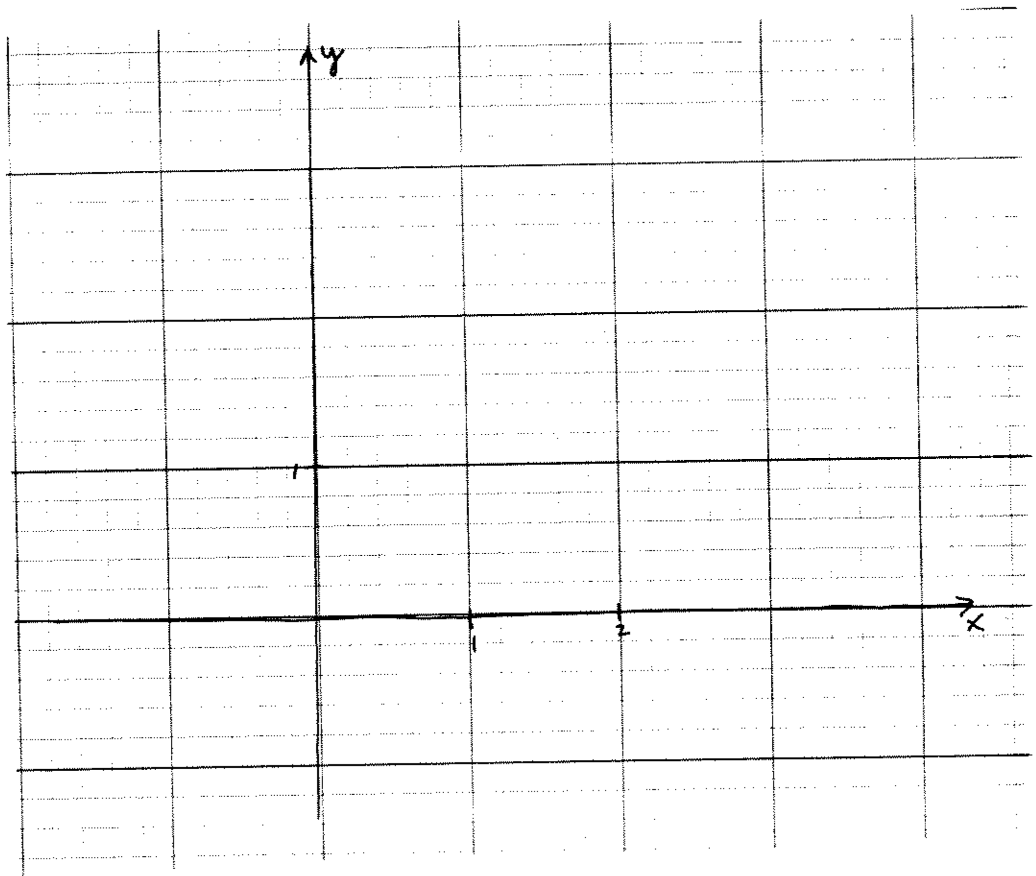
Compute and plot the points (x, y), for the indicated t-values,

for the line

$$\boxed{\begin{aligned} x &= 1 + 3t \\ y &= 2 + t \end{aligned}}$$



inputs
(domain)



outputs
(range)

For any point P, the vector $\overrightarrow{OP}$ from the origin to P is called the position vector of P
We often write a parametric line (or curve) by expressing a formula for the position vector $\vec{F}(t)$, e.g.

$$\boxed{\vec{F}(t) = \langle 1, 2 \rangle + t \langle 3, 1 \rangle}$$

for the line in example 2.
We call $\langle 3, 1 \rangle$ the (a) direction vector for this particular line

Example 3 Add the four position vectors for the points you found in 2
(Of course, usually when we sketch the line we just sketch the endpoints of the position vectors!!)

Example ④ : Find the slope-intercept form of the line in ② by solving simultaneously for t in each equation.

Lines in $\mathbb{R}^3$

given parametrically by

$x = x_0 + at$
$y = y_0 + bt$
$z = z_0 + ct$

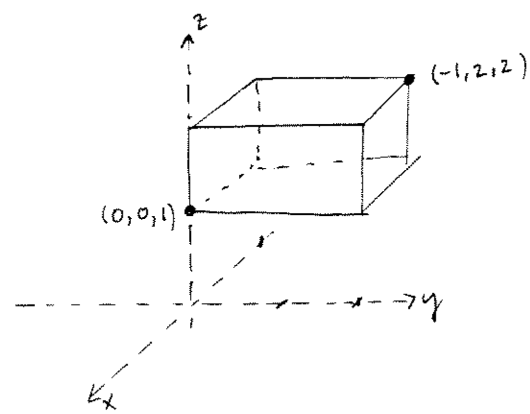
or, by expressing the position vector

$$\vec{r}(t) = \langle x_0, y_0, z_0 \rangle + t \langle a, b, c \rangle$$

a point on the line ↑ ↑ direction vector

Example ⑤

Find a parametric expression for the line thru $P = (0,0,1)$ and $Q = (-1,2,2)$



Symmetric form for a line:
Solve for t simulataneously in parametric form:

$$(t =) \quad \underbrace{\frac{x-x_0}{a} = \frac{y-y_0}{b}}_{\text{eqtn of a plane}} \; \frac{}{} $$

$$\frac{y-y_0}{b} = \frac{z-z_0}{c} \quad \text{(eqtn of another plane)}$$

this is expressing the line as an intersection of 2 planes (well, actually of 3 planes)

Example ⑥ : Find symmetric eqtns of the line in ⑤.

Example ⑦ : Find parametric eqtns for the line $\frac{x-3}{2} = \frac{y-5}{3} = \frac{z+1}{7}$

Example 8

Find symmetric and parametric equations for the line of intersection between the two planes

$$x - y + 2z = 5$$
$$2x + y + z = 1$$

Lots of ways to do this problem!