Math 2210-4
Wed 19 Jan

remember optional problem session for Fri HW
tomorrow (Thurs) AEB 310, 11:50-12:40

Last week:

$\mathbb{R}^3$

vectors

geometric
algebraic
+
scalar mult
dot product
definition : $\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$ $\quad (\vec{a} = \langle a_1, a_2, a_3\rangle,\ \vec{b} = \langle b_1, b_2, b_3\rangle)$
geometric formula : $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

- continue discussion, page 2 Friday notes.
& page 3

Applications of dot product

(A) Use geometric formula to find $\cos\theta$ (or θ); did examples Fri. Also, see HW

(B) Planes in $\mathbb{R}^3$:

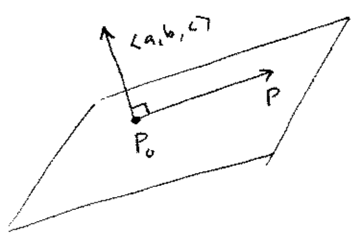
Why the graph of $ax + by + cz = d$ is a plane in $\mathbb{R}^3$
and the importance of $\langle a, b, c\rangle$:

Let $P_0 = (x_0, y_0, z_0)$ be a soltn of our eqtn
Let $P = (x, y, z)$ be any soltn of our eqtn

$$\left.\begin{aligned} ax_0 + by_0 + cz_0 &= d \\ ax + by + cz &= d \end{aligned}\right\} \quad ax + by + cz = ax_0 + by_0 + cz_0$$

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

$$\langle a, b, c\rangle \cdot \underbrace{\langle (x-x_0), y-y_0, z-z_0\rangle}_{\overrightarrow{P_0P}} = 0$$



geometrically describes a plane through P_0
with normal ($\perp$) vector $\langle a, b, c\rangle$!

i.e. the plane is the set of
all $P = (x, y, z)$ so that
$\langle a, b, c\rangle \perp \overrightarrow{P_0P}$!

1. Example

   (a) Sketch the plane

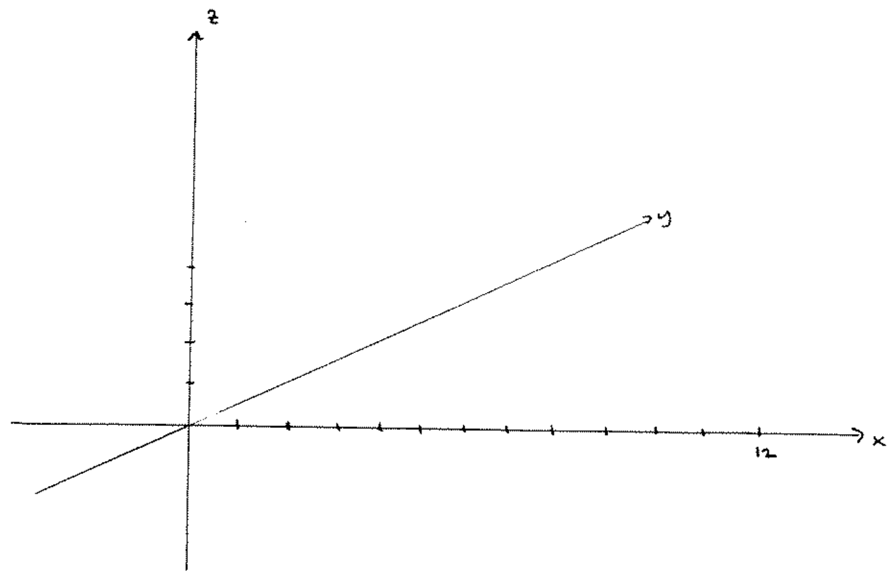
   $x + 3z = 12$ in x-y-z space.

   Include a normal vector

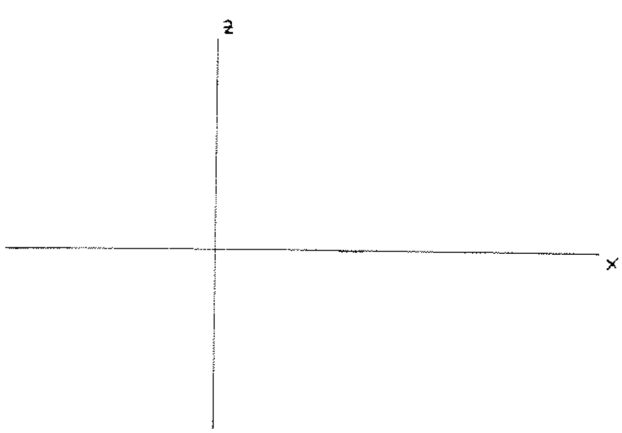
   (b) Add the plane $x = 8$

   (c) What is the (smaller of the two) angle(s) between these two planes?

   (notice it's the angle between the two plane normal vectors!)



cross section in a plane ⊥ to both planes:

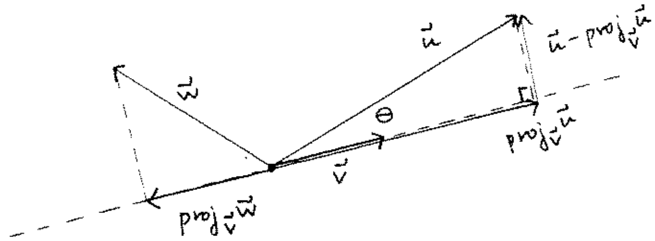


Remark: you could redo lines in $\mathbb{R}^2$ with the dot product. Also, $(n-1)$-dimensional hyperplanes in $\mathbb{R}^n$

C Projections

the (vector) projection of $\vec{u}$ onto $\vec{v}$ $\mathrm{pr}_{\vec{v}}\vec{u}$ (or $\mathrm{proj}_{\vec{v}}\vec{u}$)

is the unique scalar multiple of $\vec{v}$ so that $\vec{u}-\mathrm{proj}_{\vec{v}}\vec{u}$ is $\perp$ to $\vec{v}$:



$$\mathrm{proj}_{\vec{v}}\vec{u} = \underbrace{|\vec{u}|\cos\theta}_{\text{dist}}\ \underbrace{\frac{\vec{v}}{|\vec{v}|}}_{\text{unit vect in } \vec{v} \text{ dir}}$$

$$= \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}\ \frac{\vec{v}}{|\vec{v}|}$$

$$\boxed{\begin{aligned}\mathrm{proj}_{\vec{v}}\vec{u} &= \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\,\vec{v} \\ \mathrm{comp}_{\vec{v}}\vec{u} &= \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}\end{aligned}}$$

the scalar projection of $\vec{u}$ onto $\vec{v}$ (also called the component of $\vec{u}$ in the $\vec{v}$ direction) $\mathrm{comp}_{\vec{v}}\vec{u}$

is $|\vec{u}|\cos\theta$,

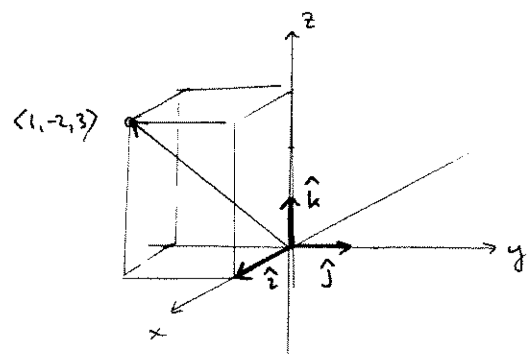
i.e. the signed length of $\mathrm{proj}_{\vec{v}}\vec{u}$

② Example

$\vec{u} = \langle 1,-2,3\rangle$

What are the (vector) projections of $\vec{u}$ in the $\hat{\imath},\hat{\jmath},\hat{k}$ directions?

What are the corresponding components?



Example

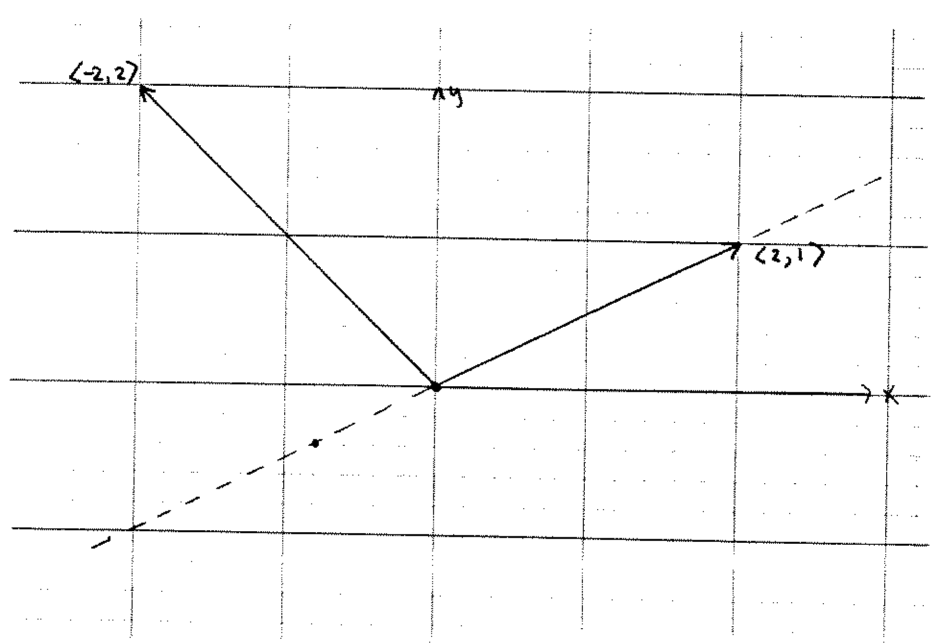
③ Let $\vec{u} = \langle -2,2\rangle$

$\vec{v} = \langle 2,1\rangle$

Compute

$\mathrm{proj}_{\vec{v}}\vec{u}$, $\mathrm{comp}_{\vec{v}}\vec{u}$.

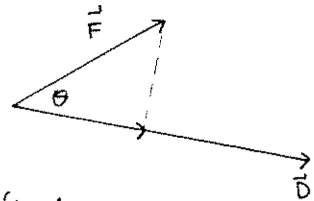
Illustrate.



Application

(D) Work :

If $\vec{F}$ is a constant force field
and an object is displaced by the vector $\vec{D}$



Then work W is defined
by $W = \underbrace{(\text{comp}_{\vec{D}} \vec{F})}_{\frac{\vec{F}\cdot\vec{D}}{|\vec{D}|}} |\vec{D}|$ (how we did work in 1210-1220, using scalar computations)

so $\boxed{W = \vec{F}\cdot\vec{D}}$ This is the work done by the field,
which is opposite the work done by the object

Example (4)

A 10 kg mass is subject to
a force of 10g = 98 Newtons,
pointing vertically down (from gravity)
i.e. $\vec{F} = \langle 0,0,-98\rangle$
How much work is done (by the field)
in moving ~~the~~ object from $\langle 1,-2,3\rangle$ to $\langle 100,0,103\rangle$?
(distance measured in meters)