Math 2210-4
Wed 12 Jan

§13.2, 13.3, 14.2 (continued Friday)

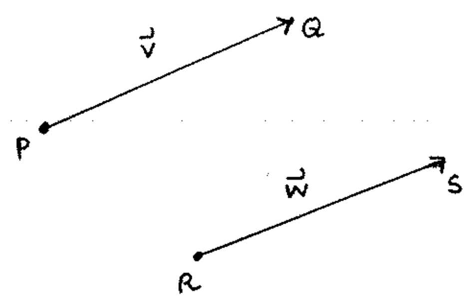
Vectors

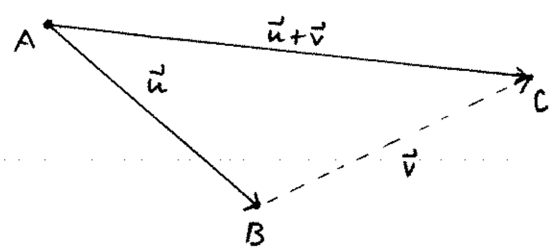
(actually, we should first discuss last page of Monday's notes).

<u>vector</u> (<u>geometric def.</u>) an "arrow", or directed line segment, (in $\mathbb{R}^2$, $\mathbb{R}^3$, or $\mathbb{R}^n$) characterized by its <u>direction</u> and its <u>length</u> (or <u>magnitude</u>)

vectors are <u>equivalent</u> (or <u>equal</u>) if they have the same length and direction.

If the vector $\vec{v}$ starts at P and ends at Q we write $\vec{v} = \vec{PQ}$



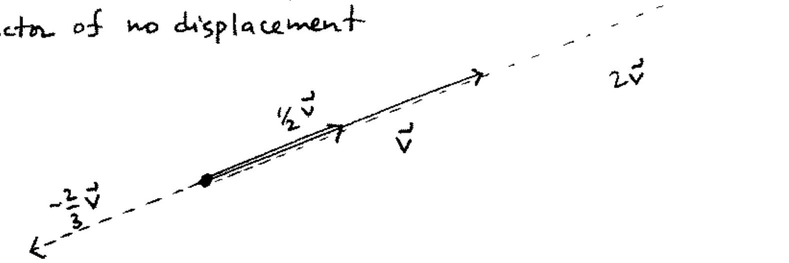


① which vectors above look like they might be equal?

<u>Vector addition</u> $\vec{u}+\vec{v}$ is the vector obtained by placing the vector $\vec{v}$ so that its initial point is at the terminal point for $\vec{u}$. Then $\vec{u}+\vec{v}$ is the vector from the starting point of $\vec{u}$ to the terminal point of $\vec{v}$.
(So $\vec{u}+\vec{v}$ measures net displacement obtained by first displacing by $\vec{u}$, and then by $\vec{v}$.)

<u>scalar multiplication</u> (scalar is a synonym for real number).

if $c > 0$ then $c\vec{v}$ has length $|c|$ $(=c)$ times length of $\vec{v}$, and points in same direction
if $c < 0$ then $c\vec{v}$ has length $|c|$ times length of $\vec{v}$, but points in opposite dir.
if $c = 0$ then $c\vec{v} = \vec{0}$, the vector of no displacement



Some pictures in $\mathbb{R}^2$. (We could do analogous sketching in $\mathbb{R}^3$)

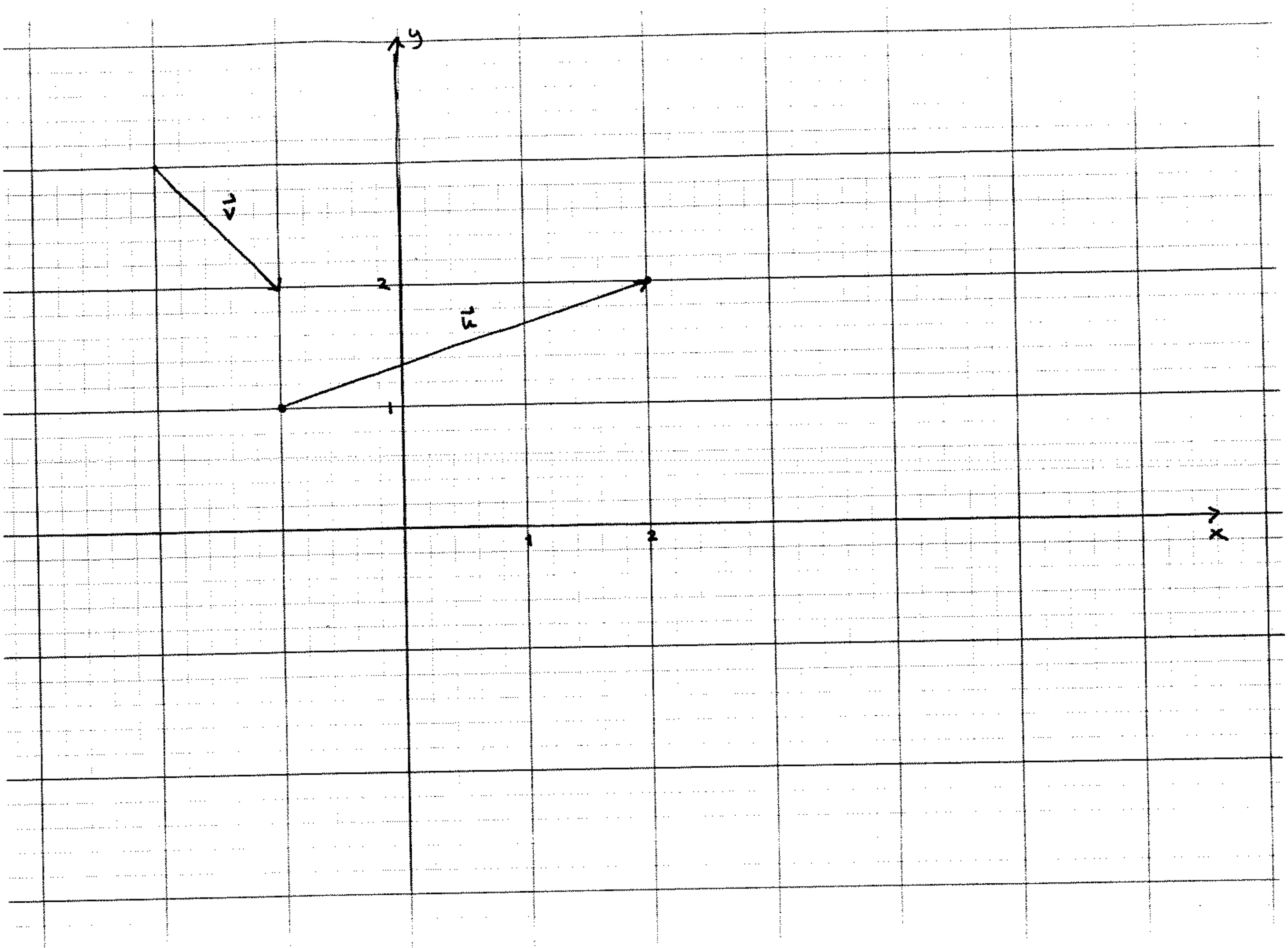
(2) Given $\vec{u}, \vec{v}$ as drawn below. Find

a) $|\vec{u}|$ (this is the notation for magnitude (length) of $\vec{u}$)

b) $\vec{u}+\vec{v}$
$\vec{v}+\vec{u}$

c) $-2\vec{u}$

d) $\vec{u}-\vec{v}$ (means $\vec{u}+(-\vec{v})$)



Notice that knowing vector is the same as knowing its displacement in each coordinate direction

<u>vector</u> <u>(algebraic definition)</u>

$\vec{a} = \langle a_1, a_2 \rangle$ corresponds to a geometric vector in the plane with displacements a_1 in x-dir, a_2 in y-dir

$\vec{a} = \langle a_1, a_2, a_3 \rangle$ corresponds to a geometric vector in $\mathbb{R}^3$, with displacements a_1, a_2, a_3 in x-y-z dirs, respectively

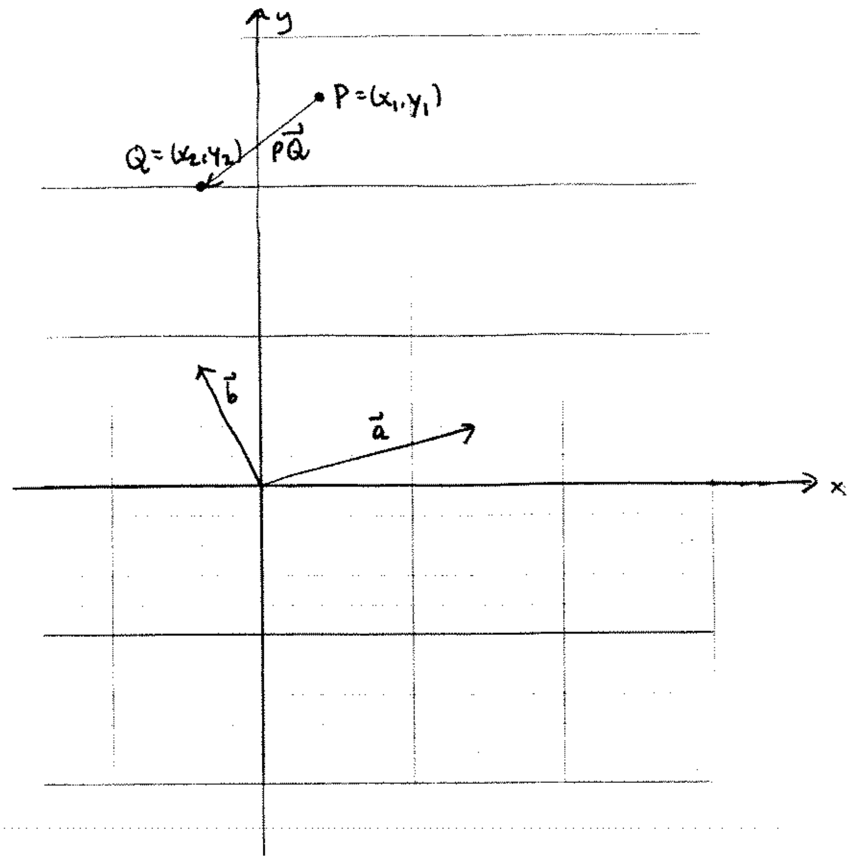
etc.

the a_1, a_2 (a_3) are called the <u>components</u> of $\vec{a}$ in x, y, (z) dirs

(3) Express $\vec{u}, \vec{v}, \vec{u}+\vec{v}, -2\vec{u}, \vec{u}-\vec{v}$ algebraically.

(4) If $P = (1,4,2)$ Express $\overrightarrow{PQ}$ algebraically.
$Q = (0,3,3)$

(5) Same question for
$P = (x_1, y_1)$
$Q = (x_2, y_2)$



(6) If $\vec{a} = \langle a_1, a_2 \rangle$
$\vec{b} = \langle b_1, b_2 \rangle$

What is

$\vec{a} + \vec{b} =$

$c\vec{a} =$

$\vec{a} - \vec{b} =$

(7) Same question if
$\vec{a} = \langle a_1, a_2, a_3 \rangle$
$\vec{b} = \langle b_1, b_2, b_3 \rangle$.

<u>Vector arithmetic</u> : You should be able to check these properties!

magnitude: if $\vec{a} = \langle a_1, a_2 \rangle$, $|\vec{a}| = \sqrt{a_1^2 + a_2^2}$
if $\vec{a} = \langle a_1, a_2, a_3 \rangle$, $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$
if c is a scalar,
$$|c\vec{a}| = \underset{\substack{\uparrow \\ \text{abs val.}}}{|c|}\,\underset{\substack{\uparrow \\ \text{mag.}}}{|\vec{a}|}$$

(i) $\vec{a} + \vec{b} = \vec{b} + \vec{a}$

(ii) $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$

(iii) $\vec{a} + \vec{0} = \vec{a}$ $\quad (\vec{0} = \langle 0,0 \rangle$ or $\langle 0,0,0 \rangle)$

(iv) $\vec{a} + (-\vec{a}) = \vec{0}$

(v) $c(\vec{a} + \vec{b}) = c\vec{a} + c\vec{b}$

(vi) $(c+d)\vec{a} = c\vec{a} + d\vec{a}$

(vii) $(cd)\vec{a} = c(d\vec{a})$

(viii) $1\vec{a} = \vec{a}$

(8) Check some of the vector arithmetic properties on page 3 using algebra (this will make you comfortable using them).
Illustrate with pictures. (geometry)

<u>Unit vectors</u> : A vector with magnitude 1 is called a unit vector

if $\vec{a}$ is a vector, then $\underbrace{\frac{1}{|\vec{a}|}}_{\text{scalar factor}}\vec{a}$ is a unit vector in same direction!

Since $\left|\frac{1}{|\vec{a}|}\vec{a}\right| = \frac{1}{|\vec{a}|}|\vec{a}| = 1$!

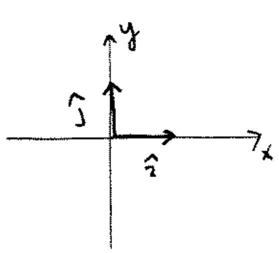
see page (3).

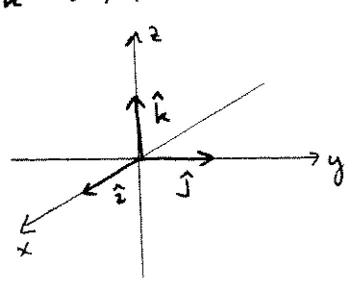
(9) Find a unit vector in direction of $\vec{a} = \langle 3, 4\rangle$

Important unit vectors : $\hat{\imath}, \hat{\jmath}, \hat{k}$

$\hat{\imath} = \langle 1, 0\rangle$
$\hat{\jmath} = \langle 0, 1\rangle$ in $\mathbb{R}^2$

$\hat{\imath} = \langle 1, 0, 0\rangle$
$\hat{\jmath} = \langle 0, 1, 0\rangle$ in $\mathbb{R}^3$
$\hat{k} = \langle 0, 0, 1\rangle$





(10) Express $\langle 3, -1, 4\rangle$ as a <u>linear combination</u> (i.e. a sum of scalar multiples) of $\hat{\imath}, \hat{\jmath}, \hat{k}$. Draw picture.