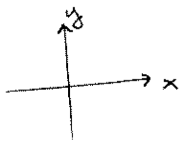
Math 2210-4

Monday 10 January

§14.1 Three-space $\mathbb{R}^3$

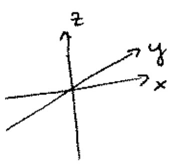
1210-1220: $\mathbb{R}^2$

"x-y plane"

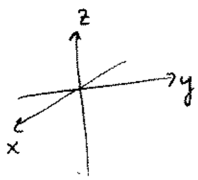


2210: mostly $\mathbb{R}^3$

"x-y-z space"



or



all axes are chosen perpendicular to eachother.

we choose <u>right-handed</u> ways of labeling our three directions (as opposed to left-handed)

or

curling from x to y axis.

(We had an analogous convention in $\mathbb{R}^2$)

If $P = (x_1, y_1, z_1)$ in $\mathbb{R}^3$

that means we have displaced from the origin by amts

x_1 in x-dir

y_1 in y-dir

z_1 in z dir.

<u>eight</u> <u>octants</u> in 3-space.

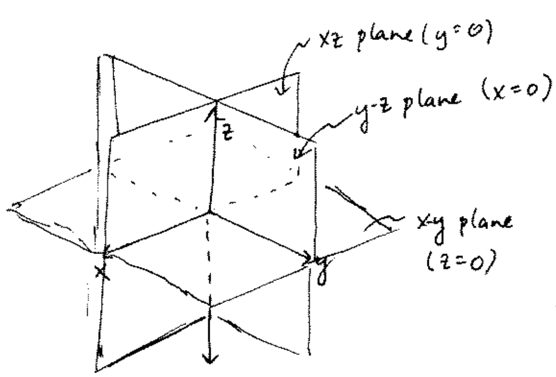
usually only specify 1st octant → where

$x > 0$

$y > 0$

$z > 0$

all hold.

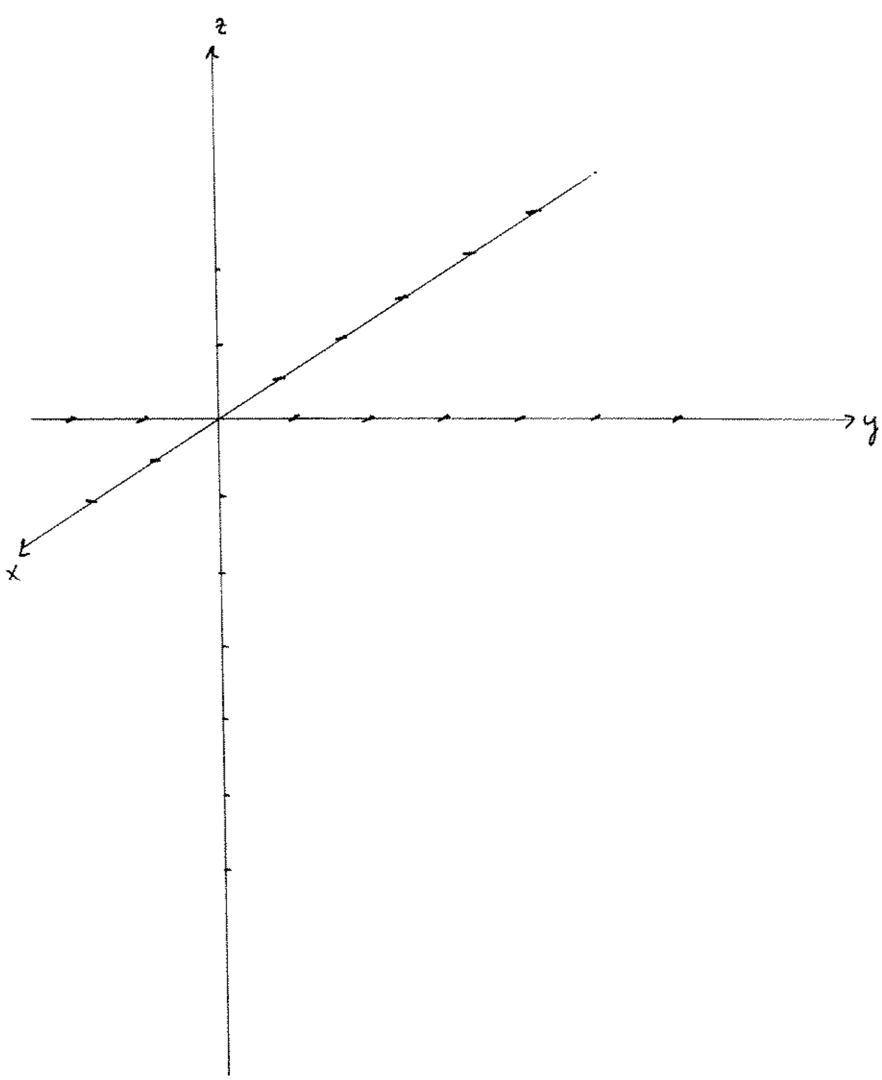


Examples

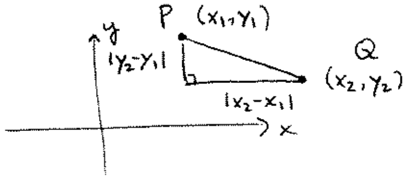
1) plot the point
$P = (-4, 3, -5)$
as well as the rectangular coord box which has P and the origin as opposite corners.

2) Use equalities and inequalities to specify
   (a) the region inside the box
   (b) two different rectangular faces of the box
   (c) two different edges of the box.

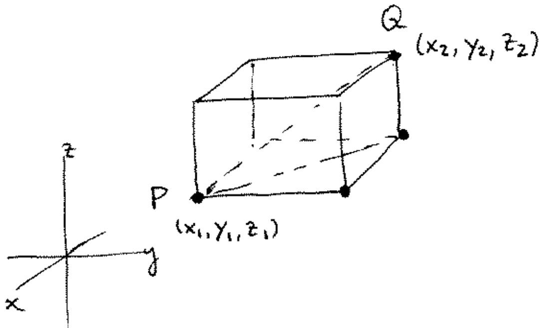
3) Try to figure out the straight line distance from the origin to P.



Distance formulas



$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Pythagorean Thm!



$d = ?$ Use Pythag twice!

Example (4) : Identify and sketch the points which satisfy

$$x^2 + y^2 + z^2 - 10x - 8y - 12z + 68 = 0$$ (We call this "graphing the equation")

hint: complete the square first.

Examples

(5) graph the equation $x^2+y^2=1$

(6) graph the region $1 \le x^2+y^2 \le 4$

(7) graph the plane $x+2y=4$

(8) graph (a piece of) the plane

$$x+2y+3z=6$$

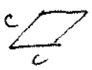
(9) graph (a piece of) the plane

$$x+2y+3z=0$$

(10) graph the equation $z=y^2$

Homework Set #1 : Circled problems are to be handed in.
Due Friday 14 Jan. Others are recommended but not required.
(This week they're all circled)

(1) The Pythagorean Theorem says that for a right Δ, with legs a, b & hypotenuse c, $c^2 = a^2 + b^2$

Use the following diagram (by computing the area of the $(a+b) \times (a+b)$ square two ways) to prove the Pythagorean Thm. Hint: first show that the inside  is a square, using the fact that the sum of angles in a triangle is 180°

(2) Consider the point $P = (1, -2, 3)$.

(a) Draw the x-y-z axes as on page 2 of today's (1/10) notes, and then draw the coordinate box for P, as we did on page 2. (So P & the origin are opposite vertices.)

(b) Use inequalities to specify the region inside the box

(c) Use inequalities and the equality $x = 1$ to specify the "front" face of the box

(d) Use equalities and inequalities to specify (separately) the three edges which contain the point $(1, -2, 3) = P$

(e) How far is it from P to the origin?

(f) " " " " to the x-y plane?

(g) " " " " to the x-axis?

(3) Sketch pieces of the following planes (so that the viewer can visualize the entire plane)

(a) $2x + 6y + 3z = 12$ (this is 14.1 #17)

(b) $z = 2$ (14.1 #29a)

(c) $2x - y - z = 0$

(4) sketch (graph) the following

(a) $x^2 + y^2 + z^2 = 9$ (14.1 #23)

(b) $x^2 + y^2 + z^2 \leq 9$

(c) $x^2 + y^2 = 4$ (14.1 #29c)

(d) $x^2 + y^2 \leq 4$

(5) Show that $(4,5,3)$, $(1,7,4)$, $(2,4,6)$ are vertices of an equilateral triangle.
Hint: distance formula (14.1 #6)