Math 2210-4
Fri 25 Feb

limits and continuity

generalize what you did in Calc.

Hw for Fri Mar 4

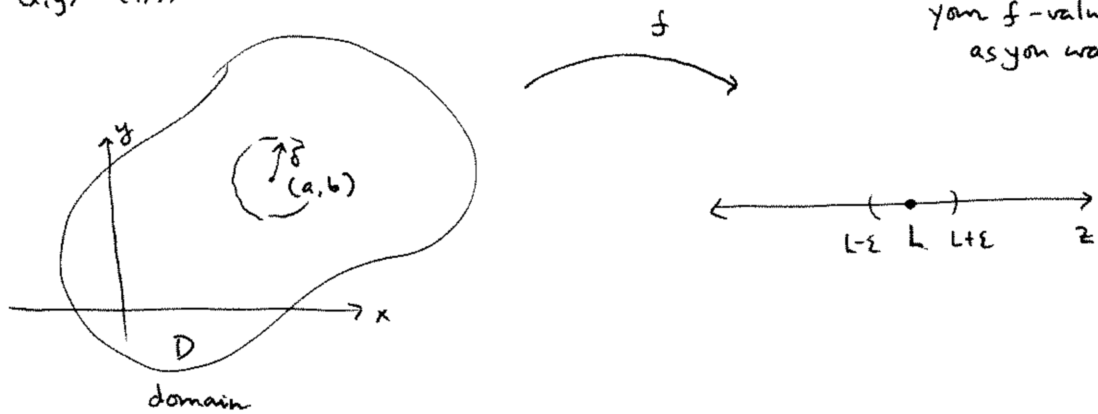
§15.3 1, 2, 3, 5, 7, 8, 10, 11
27 abc 28
§15.4 2, 9 11, 12 15 17
§15.5 1, 9 14 15 17 19 20, 24 25 31

$f: D \to \mathbb{R}$, $D \subset \mathbb{R}^n$

$f(x,y)$
$f(x,y,z)$

etc.

$\lim_{(x,y)\to(a,b)} f(x,y) = L$ means: Should mean: if you are close enough to (a,b) in the domain, then your f-value is as close as you want to L



precise def: for all $\varepsilon > 0$ there exists a $\delta > 0$ so that

$|\langle x,y\rangle - \langle a,b\rangle| < \delta$
$\Rightarrow |f(x,y) - L| < \varepsilon$

What would this mean in terms of the graph $z = f(x,y)$?

Examples Find

$\lim_{\langle x,y\rangle \to \langle 2,0\rangle} 3x$

$\lim_{\langle x,y\rangle \to \langle 2,0\rangle} 4y$

$\lim_{\langle x,y\rangle \to \langle 2,0\rangle} e^{3x}(4y^2+1)$

A function $f(x,y)$ is <u>continuous</u> at (a,b) if

$$\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b).$$

We have the same limit theorems as in 1-variable Calc:

If $\lim_{(x,y)\to(a,b)} f(x,y) = L$ and $\lim_{(x,y)\to(a,b)} g(x,y) = M$ then

$f+g \to L+M$

$fg \to LM$

$f/g \to L/M$ (as long as $M \neq 0$).

So if f, g are cont (at (a,b)) then so are fg, $f+g$, f/g (if $g(a,b) \neq 0$).

Also, if $f(x,y)$ is cont at (a,b) and g is cont at $f(a,b)$ then $g \circ f$ is cont at (a,b)

etc.

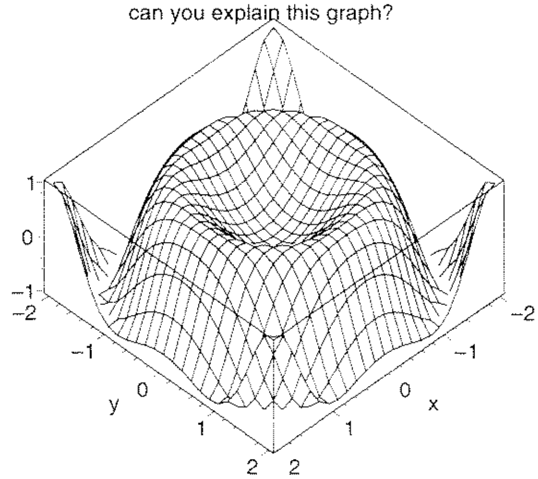
Analogous for functions of (x,y,z), (x,y,z,w) etc.

If a function is continuous, you can get the limit by "plugging in"

(1) $\lim_{(x,y)\to(0,0)} \sin(x^2+y^2)$

$z = \sin(x^2+y^2)$

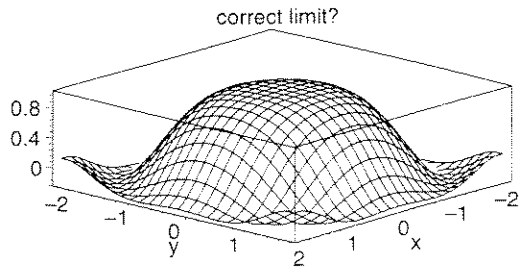
can you explain this graph?



(2) $\lim_{(x,y)\to(0,0)} \dfrac{\sin(x^2+y^2)}{(x^2+y^2)}$

$z = \dfrac{\sin(x^2+y^2)}{(x^2+y^2)}$

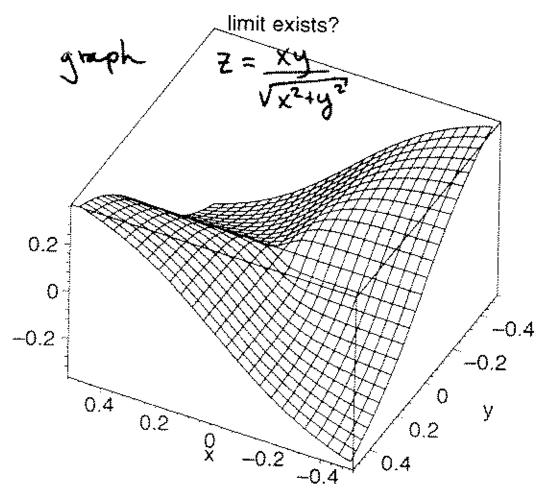
$\lim_{(x,y)\to(0,0)} \dfrac{\sin(x^2+y^2)}{x^2+y^2} = ?$



(3) $\lim_{(x,y)\to(0,0)} \dfrac{xy}{\sqrt{x^2+y^2}}$

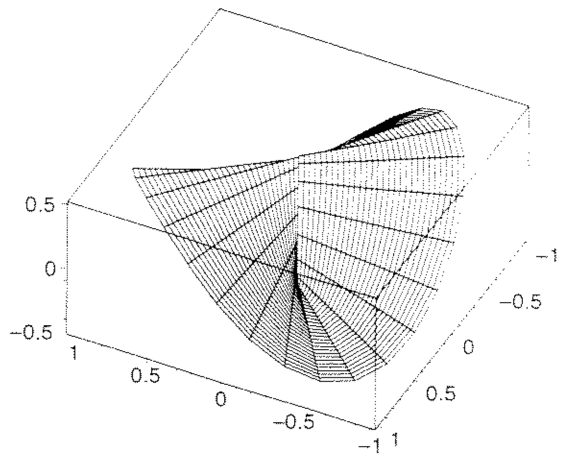
try polar coords!

graph $z = \dfrac{xy}{\sqrt{x^2+y^2}}$



(4) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$

try polar coords!
or look along lines
thru origin



(5) $f(x,y) = \begin{cases} 1 & y \geq 2x^2 \\ & \text{or } y \leq x^2 \\ 0 & \text{if } x^2 < y < 2x^2 \end{cases}$

does $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$ exist?