

Math 2210-4

Wed Feb 23

6.15.2 Partial derivatives

$$f: D \rightarrow \mathbb{R}$$

$\widehat{\mathbb{R}^n}$

$f(x, y)$
 $f(x, y, z)$
 $f(x, y, z, t)$ etc.

partial deriv. of f with respect to x , at (x_0, y_0) :

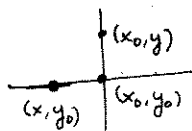
$$f_x(x_0, y_0) := \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x}$$

↑
 measures rate of change of f in x -direction.

In fact, it's just a 1D derivative

$$g'(x_0); \quad g(x) = \overset{\text{const}}{f(x, y_0)}$$

↑
variable



partial deriv. of f w.r.t. y , at (x_0, y_0) :

$$f_y(x_0, y_0) := \lim_{\Delta y \rightarrow 0} \frac{f(x_0, y_0 + \Delta y) - f(x_0, y_0)}{\Delta y}$$

↑
 measures rate of change of f in y -dir.

Is just a 1D deriv

$$h'(y_0); \quad h(y) = f(x_0, y)$$

↑
variable

So, partial derivs are easy to compute!

$$f(x, y) = e^{2x} \cos 3y$$

$$f_x =$$

$$f_y =$$

$$f_x(0, \pi) =$$

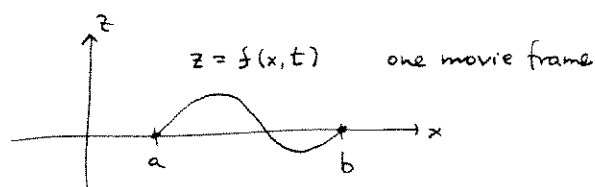
$$f_y(0, \pi) =$$

What should be the definition of $f_z(x_0, y_0, z_0)$?

Geometric and Physical interpretations

- Suppose $f(x, t)$ is the vertical displacement of a vibrating string ($a \leq x \leq b$) at time t .

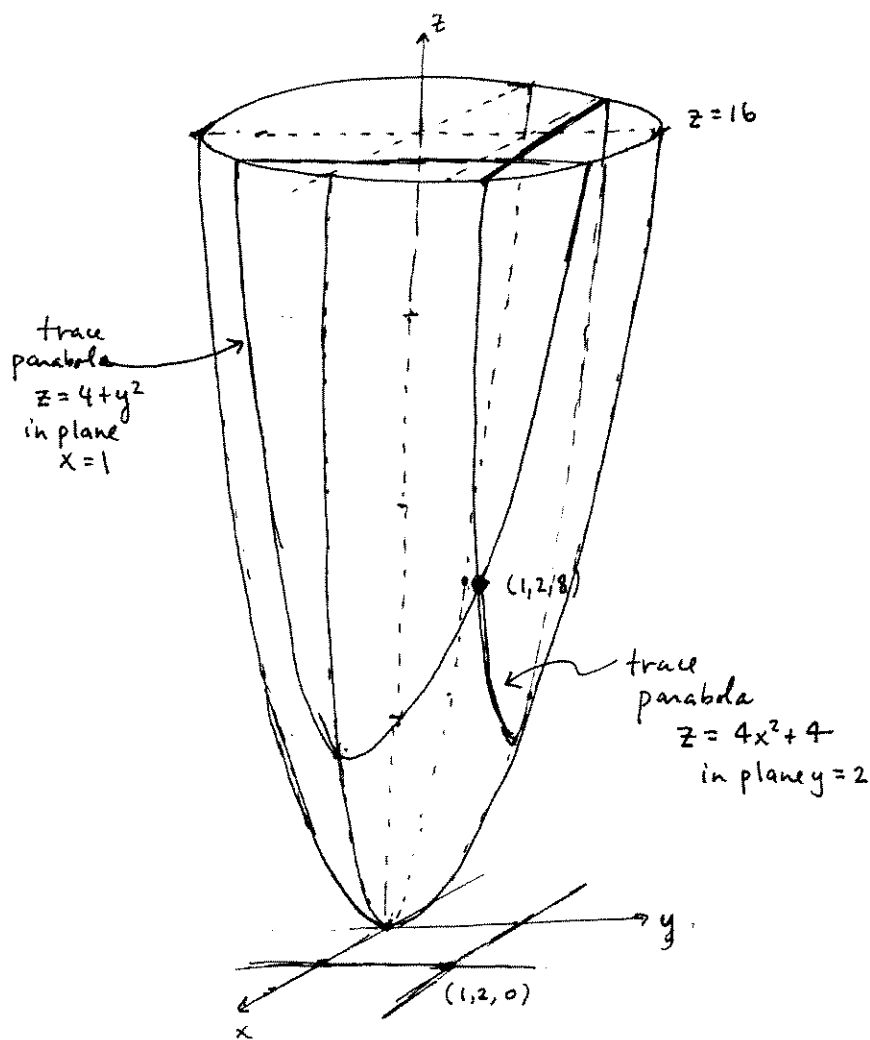
What do $f_x(x_0, t_0)$, $f_y(x_0, t_0)$ represent physically?



- Slopes of trace curves:

function $f(x, y) = 4x^2 + y^2$
graph $z = 4x^2 + y^2$

- Interpret $f_x(1, 2)$ and $f_y(1, 2)$ as slopes of trace curves.
- Can you find tangent vectors to these curves at $(1, 2, 8)$?
- Could you find the tangent plane to the paraboloid at $(1, 2, 8)$?



- Re estimate

$$f_x(1,2)$$

$$f_y(1,2)$$

from this "contour
map" of the level
curves of f ,
using an index
card and ruler.

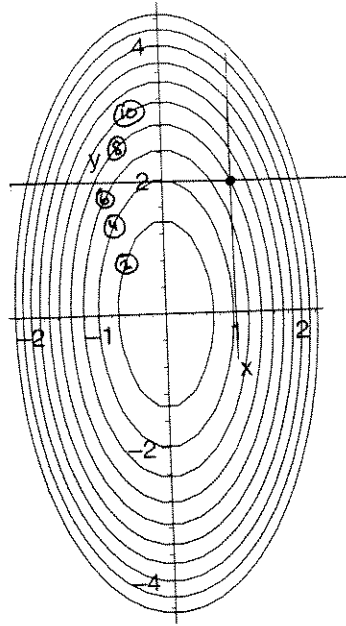
At $(1,2)$,

$$I \text{ get } f_x \approx \frac{\Delta f}{\Delta x} \approx \frac{4}{.5} = 8$$

$$f_y \approx \frac{\Delta f}{\Delta y} \approx \frac{4}{1.1} \approx 4$$

How about you?

level curves
of f , in the domain $\sim f=2, f=4, f=6$ etc.



Other notation for partial derivatives,
and higher order partial derivs

$$f_x = \frac{\partial f}{\partial x} = D_x f$$

$$f_y = \frac{\partial f}{\partial y} = D_y f$$

$$f_{xy} = (f_x)_y = \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial x} \right) = \frac{\partial^2 f}{\partial y \partial x} = D_y D_x f$$

$$f_{xxy} = ((f_x)_x)_y \text{ etc.}$$

Example : Compute all 1st and 2nd order partial derivatives
for $f(x, y, z) = z \sin(x/y) + xy^2z^3$.

What do you notice?

2210-4 Exam 1

