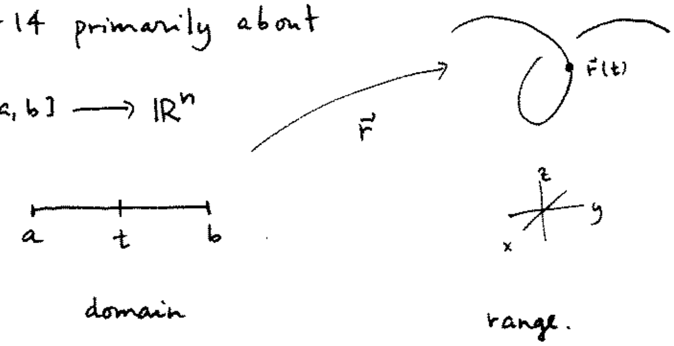
Math 2210-4
Monday 14 Feb

Exam Wed!
Practice exam sol'ns posted later today
Session to go over practice exam tomorrow
AEB 310 11:50-12:40

What happens after the exam:

Chapters 13-14 primarily about

$\vec{r} : [a, b] \longrightarrow \mathbb{R}^n$



Chapter 15 is primarily about the "reverse" situation

$$f : D \longrightarrow \mathbb{R}, \quad D \subset \mathbb{R}^n$$

(D = domain)

real examples:

$f(x,y)$ = altitude above sea level (on earth) at longitude = x, latitude = y

$T(x,y,t)$ = temperature on earth at time t

$T(x,y,z,t)$ = same, but also fcn of altitude.

hybrids

$\vec{E}(x,y,z,t)$ electric field at time t

$\vec{B}$ magnetic

$\vec{F}(x,y,z,t)$ any force field

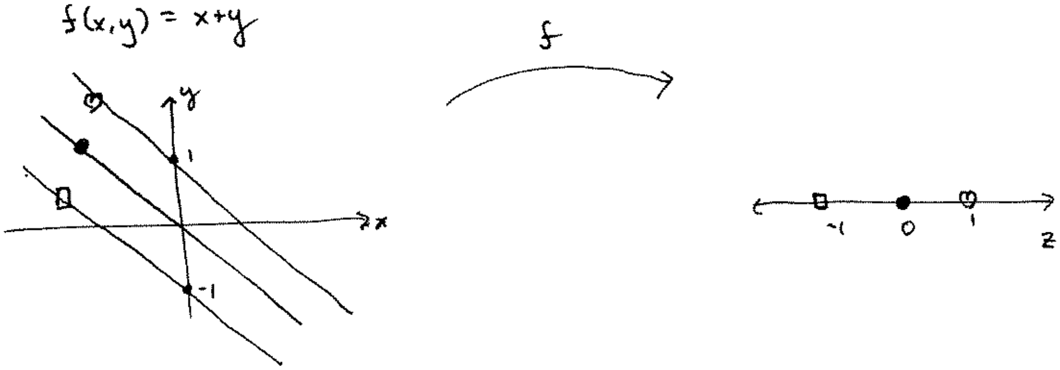
Other courses you will take soon where these ideas come up

$\vec{F}(t)$: Ordinary differential eqtns, Math 2250 (or 2280)

$f(x_1, \ldots x_n)$: Partial differential eqtns, Math 3150 (or 2280)

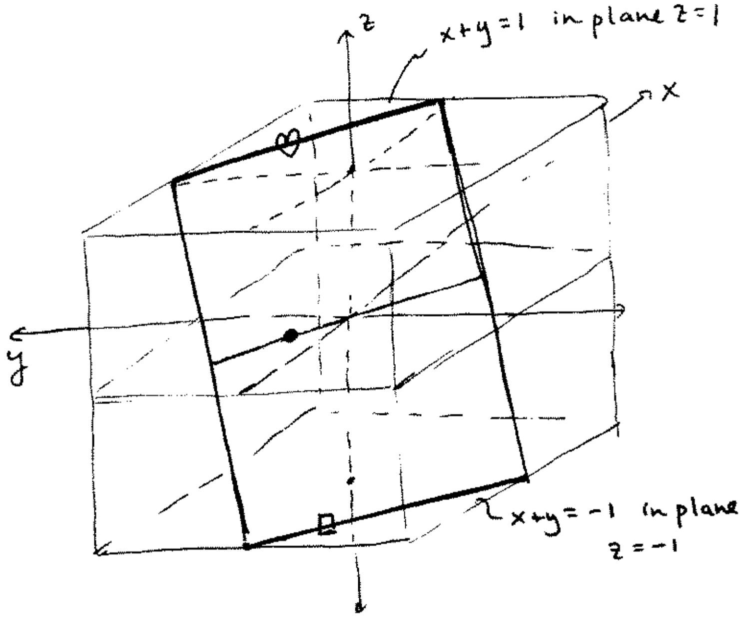
<u>Example</u>

$f(x,y) = x+y$



If the domain D is in $\mathbb{R}^2$
then you visualize the function f
by drawing the graph $z = f(x,y)$ in $\mathbb{R}^3$

Can you draw the graph
$z = x+y$?

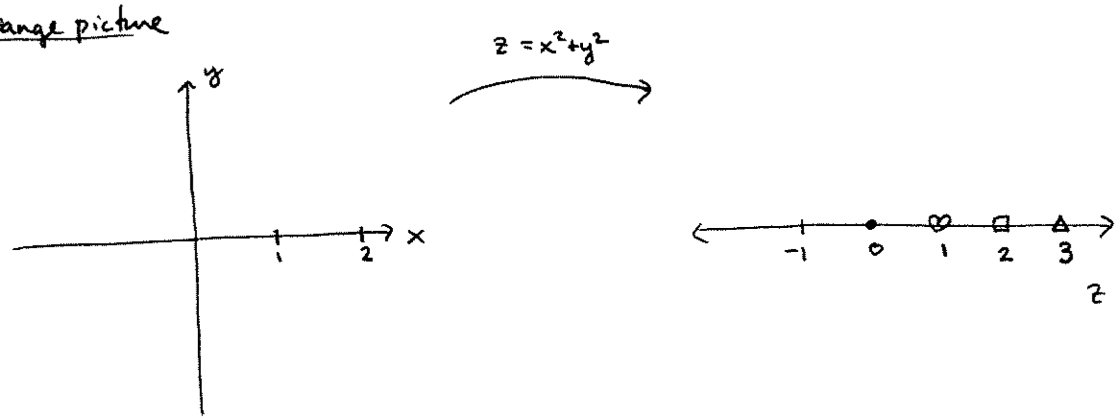


Intersections of a graph with horizontal planes (z=const)
are called <u>contours</u>

The projections of contours of a graph $z=f(x,y)$ back to the domain in the x-y plane are called <u>level curves</u>

<u>Example</u> $g(x,y) = x^2+y^2$

<u>domain-range picture</u>



<u>graph $z = x^2+y^2$</u> in $\mathbb{R}^3$

Math 2210-4
Review Sheet for first exam
February 12, 2005

Our first exam is on Wednesday February 16, in class, from 11:50-12:50. The exam will cover sections 13.1-14.6 from our text; in other words, all of chapters 13-14 except for section 14.7. In the outline below you should know each topic - have formulas memorized unless otherwise indicated! We will go through this sheet on Monday February 14 and fill in details, which is why the spacing is generous.

__Topics__

vectors and points

points in the plane and 3-space. Coordinate boxes, rectangles and line segments defined by equalities and inequalities in x,y,z

algebra and geometry of vector addition, scalar multiplication, magnitude, **i,j,k** (13.2, 13.3, 14.1)

dot product

definition (13.3, 14.2)

algebraic properties (13.3)

geometric formula (13.3, 14.2)

angle between vectors

equations of planes

finding a plane equation from various pieces of information

angle between intersecting planes

projection

components

work

distance from a point to a plane

(Could you explain this formula if reminded of it? Could you use it?)

cross product (14.3)

definition

geometric formulas

direction

magnitude

areas of parallelgrams, triangles

cross and dot product together (14.3, 14.4)

volumes of parallelepipeds

formulas to compute distance from point, line or plane to line or plane.

(Could you explain why such a formula is true, if given the formula? Could you use the formula?)

surfaces as graphs of equations in three-space (14.6)
planes, cylinders, quadric surfaces

identification of surface from its equation, i.e. from among the choices of plane, cylinder, ellipsoid, 1 and 2-sheeted hyperboloids, elliptic and hyperbolic paraboloids?

sketching a surface with the aid of trace curve sketches in coordinate (or parallel) planes.

parametric curves (13.1, 13.4, 13.5, 14.4, 14.5)
showing a parametric curve lies on the graph (solution set) of an equation.
for a plane curve relate dy/dx to the tangent vector <dx/dt, dy/dt>

position vector, velocity (tangent) vector, acceleration vector
how to compute, where to draw, geometric and physics meanings

differentiation rules for vector-valued functions (sum, products, chain)

lengths of curves
unit tangent and normal vectors (Could you find T for any curve, N at least for a plane curve? No binormal B on exam)

curvature (Do you understand how to compute curvature from the various formulas?)

circle of curvature (Could you find the circle of curvature for a plane curve?)

decomposition formula of acceleration into tangential and normal components (Do you understand this formula, if reminded of its precise form? Can you work in various ways with this formula, such as in the extended problem 36 in last hw assignment? Could you explain how this formula is derived? Could you use it to get the formula(s) for curvature?)

Name..

I.D. number..

Math 2210-4

Practice Exam 1

February 14, 2005

The exam will be closed-book and closed-note. You will be allowed the use of a scientific calculator (only), but not one which is capable of graphing or of solving linear algebra equations. You won't actually need the calculator, but there could be times when numerical values would be a hint about whether you've correctly solved a problem. **In order to receive full or partial credit on any problem, you must show all of your work** and **justify your conclusions.** There are 100 points possible. The point values for each problem are indicated in the right-hand margin. On the actual test there will be space for your work. **Good Luck!**

1) Consider the three points P = (1,2,-1), Q = (0,3,0) and R = (-2,5,1).

1a) Find a parametric formula for the line through P and Q. (5 points)

1b) Find an equation for the plane containing P,Q and R. (15 points)

1c) Find the area of the triangle having P,Q, R as vertices. (5 points)

1d Find the distance from the plane in (1a) to the point S=(1,2,3). You may use the formula

$$dist = \frac{|\vec{PS} \bullet \vec{N}|}{|\vec{N}|}$$

where P is any point on the plane, and N is a perpendicular vector to the plane. (5 points)

1e) Explain, using your understanding of the dot product and an appropriate picture, why the formula for distance in (1c) is true. (5 points)

2) Consider the surface which is the graph of the equation

$$\frac{x^2}{4} + \frac{y^2}{9} - z^2 = 1$$

2a) Sketch the trace curves of this surface in the three coordinate planes (10 points)

2b) Identify the type of this quadric surface (5 points)

2c) Sketch the surface. (5 points)

3) Consider the parametric curve with position vector given by

$$\mathbf{r}(t) = \langle \mathbf{e}^t,\ \mathbf{e}^{(-2t)} \rangle.$$

There is a sketch of part of the curve below.

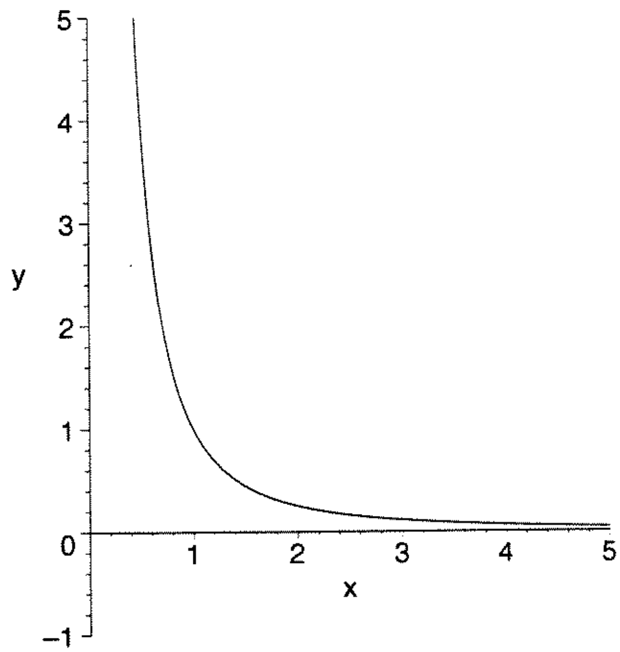
3a) Do a short computation to show that this curves lies on the graph $y = \frac{1}{x^2}$, in the x-y plane. (5 points)

3b) Compute ompute $\mathbf{r}'(t)$, $\mathbf{r}''(t)$, and the speed v(t).

(5 points)

3c) Using your favorite formula, find the curvature of this curve at the point where t=0.

(5 points)



3d) Label the point with position vector $\mathbf{r}(0)$ into the picture above. Compute, and accurately draw the vectors $\mathbf{r}'(0)$ and $\mathbf{r}''(0)$ into the picture, in the appropriate location(s), using an index card ruler. Also, compute and draw in the unit tangent and normal vectors, T and N, when $t=0$.(Hint: you can find N easily because it is perpendicular to T and the curve lies in a plane.)

(10 points)

3e) Use your sketch, an appropriate right triangle which you add to the picture, and an index card ruler to measure and record (accuracy within 0.2 suffices) the components of the acceleration $\mathbf{r}''(0)$ in the tangential and normal directions.

(5 points)

3f) Use the dot product to find the exact values of the acceleration components in the tangential and normal directions. (These should be close to your numerical approximations in (3e)!

(8 points)

3g) Refind the components in (3f) by using the roller coaster equation

$$\mathbf{r}''(t) = \left(\frac{d}{dt}\mathrm{v}(t)\right)T + \kappa v^2 N$$

where the scalar v represents speed and κ is the curvature. Note that you've already done most of the computations you need in (3b), (3c).

(7 points)