

Math 2210-4
Fri 2/11

§ 14.7 : Cylindrical and spherical coordinates in $\mathbb{R}^3$.

First recall polar coords in the plane :

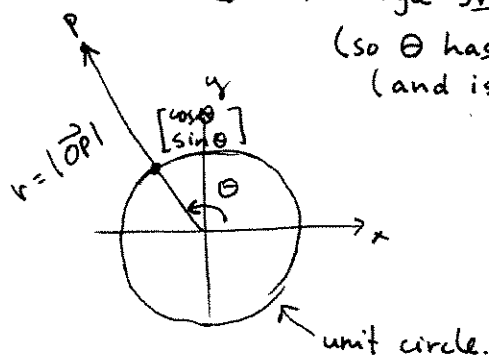
For $P = (x, y) \in \mathbb{R}^2$

$$r := \sqrt{x^2 + y^2} = \text{dist}(P, O)$$

$\theta :=$ the angle from the x-axis to $\vec{OP}$

(so θ has a sign)

(and is uniquely defined up to a multiple of 2π)



$$r = \sqrt{x^2 + y^2}$$

$$\theta = \begin{cases} \arctan y/x & \text{quadrant I, IV } (-\pi/2 \leq \theta \leq \pi/2) \\ \arccos \frac{x}{r} & \text{I, II } (0 \leq \theta \leq \pi) \\ \arcsin \frac{y}{r} & \text{I, IV } (-\pi/2 \leq \theta \leq \pi/2) \\ \arctan y/x + \pi & \text{quadrant III} \\ \text{etc.} & (\pi \leq \theta \leq 3\pi/2) \end{cases}$$

examples:

if $r=5$ $\theta=\pi/3$ $\begin{bmatrix} x \\ y \end{bmatrix} =$

if $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -10 \\ 10 \end{bmatrix}$, $\begin{bmatrix} r \\ \theta \end{bmatrix} =$

draw the curve implicitly defined by $r=3$

draw the curve implicitly defined by $\theta = \frac{2}{3}\pi$ ($r > 0$).

Why are these coords called "polar coords"?

HW for Fri 2/25
(lots more to come!)

§ 14.7 2a, 3a, 4a, 5a, 6a

7, 8, 13, 15, 9, 17, 23, 27
35, 36

Exam Wednesday!

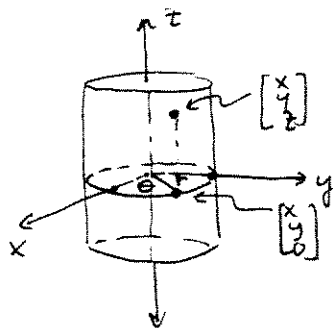
13.1-14.6

I will post a review sheet & practice exam by Saturday afternoon. Monday will be partly a review day, there will be a problem session Tuesday 11:50-12:40 AEB 310 (using problem session room)

$\mathbb{R}^3$: cylindrical coordinates:

use $\begin{bmatrix} r \\ \theta \end{bmatrix}$ in the x - y plane, keep z

i.e. $\begin{bmatrix} r \\ \theta \end{bmatrix}$ } polar coords
 $\begin{bmatrix} z \end{bmatrix}$ } z



examples: What are the $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ ("rectangular") coords of a point with cylindrical coords $\begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}$?

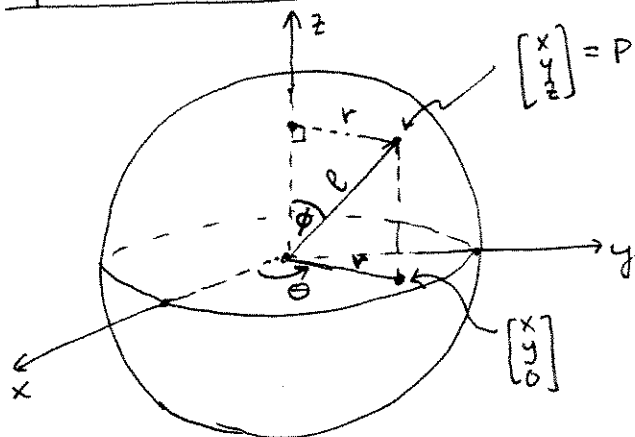
if $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ 4 \end{bmatrix}$ what are $\begin{bmatrix} r \\ \theta \\ z \end{bmatrix}$?

Sketch the surface implicitly defined by $r=3$. Why are these coords called cylindrical coords?

Sketch the surface implicitly defined by $\theta = \pi/4$ ($\& r > 0$)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} r \cos \theta \\ r \sin \theta \\ z \end{bmatrix}$$

spherical coordinates



$$\rho := \sqrt{x^2 + y^2 + z^2}$$

$\phi :=$ angle between ^{pos} z axis ($\hat{k}$) and $\vec{OP}$

$$0 \leq \phi \leq \pi \rightarrow \text{so } \phi = \arccos\left(\frac{z}{\rho}\right)$$

$\Theta :=$ polar coord angle of projection $\begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$

so (see triangles in picture)

$$z = \rho \cos \phi$$

$$r = \rho \sin \phi \quad (\text{polar } r)$$

$$\text{so } x = r \cos \Theta = \rho \sin \phi \cos \Theta$$

$$y = r \sin \Theta = \rho \sin \phi \sin \Theta$$

examples

if $\begin{bmatrix} \rho \\ \phi \\ \theta \end{bmatrix} = \begin{bmatrix} 4 \\ \frac{5}{6}\pi \\ \frac{1}{3}\pi \end{bmatrix}$, what is $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$?

if $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$, what is $\begin{bmatrix} \rho \\ \phi \\ \theta \end{bmatrix}$?

sketch the surfaces implicitly defined by

(1) $\rho = 3$;

(2) $\theta = \pi/4$;

(3) $\phi = \pi/3$;

$$\begin{aligned} x &= \rho \sin \phi \cos \theta \\ y &= \rho \sin \phi \sin \theta \\ z &= \rho \cos \phi \end{aligned}$$

other examples:

What surface is implicitly defined
by the spherical coord eqn

$$\rho = 2 \cos \phi?$$

(hint: convert to rectangular coords)

How would you find the great-circle distance between 2 points
on Earth, if you knew their latitude & longitude?

- Example 7 p. 626 & HW.