Math 2210-4
Friday April 8

HW for Fri 4/15
§16.8 3, 5, 7, 12, 15, 22, 23
§17.1 1, 2, 10, 11, 15, 17, 20 abc, 21 → book typo! they mean $f = -c/|\vec{r}|$ not $f = -c/|\vec{r}|^3$ 25
§17.2 1, 4, 7, 8, 15, 16, 17, 21

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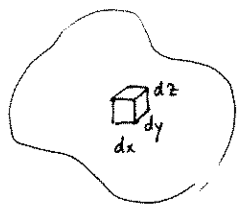
§14.7 Triple integrals in spherical & cylindrical coords

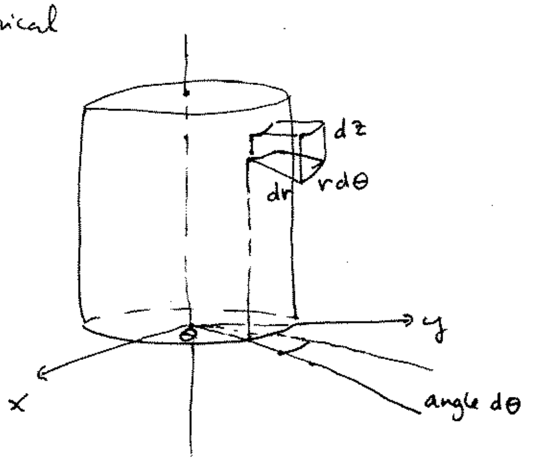
rectangular:



$dV = (dx)(dy)(dz)$

but you can partition a region using other coord systems, just like we used polar coords for double integrals

cylindrical coords



from polar coords ↓

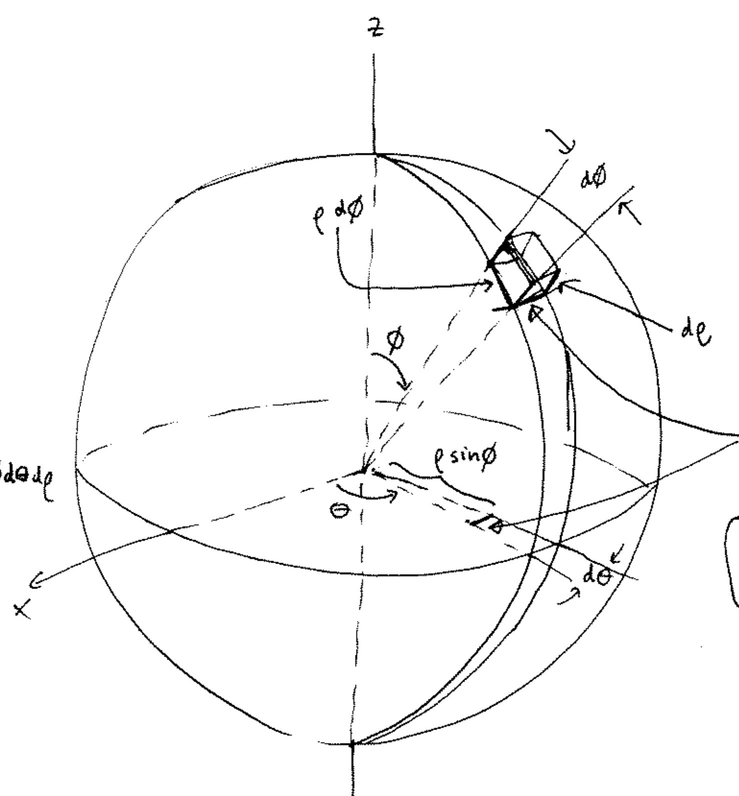
$dV = (dA)\,dz$

$$\boxed{dV = r(dr)(d\theta)(dz)}$$

so $\displaystyle\iiint_R f(x,y,z)\,dV = \iiint f(r\cos\theta, r\sin\theta, z)\, r\,dr\,d\theta\,dz$

R specified in cylind. coords

spherical coords



$? = \rho \sin\phi \, d\theta$

$$\boxed{dV = \rho^2 \sin\phi \, d\phi\, d\theta\, d\rho}$$

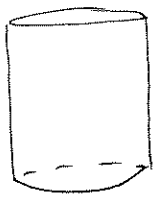
So
$\displaystyle\iiint_R f(x,y,z)\,dV$

$= \displaystyle\iiint f\begin{pmatrix}\rho\cos\theta\sin\phi\\ \rho\sin\theta\sin\phi\\ \rho\cos\phi\end{pmatrix}\rho^2\sin\phi\; d\phi\, d\theta\, d\rho$

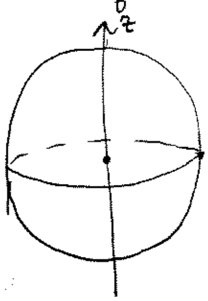
R specified in spherical coords

Examples

Volume of cylinder of radius a and height h :



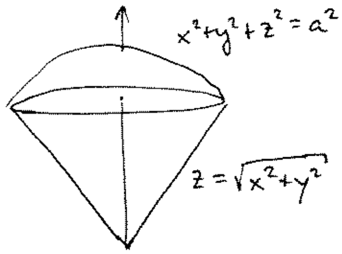
Volume of ball of radius a



Example : Our friend the ice cream cone from old HW:

region inside $x^2+y^2+z^2=a^2$

and the ice-cream cone $z=\sqrt{x^2+y^2}$



Find volume & centroid ($\delta \equiv 1$).

ans $Vol = \frac{2}{3}\pi a^3 \left(1-\frac{1}{\sqrt{2}}\right)$.

$\bar{z} = \frac{3}{8(2-\sqrt{2})} a$

```
> Vol:=int(int(int(rho^2*sin(phi),phi=0..Pi/4),rho=0..a),theta=0..2*
  Pi);
```

$$Vol := \frac{2\left(-\frac{\sqrt{2}}{2}+1\right)a^3\pi}{3}$$

```
> Mxy:=int(int(int(rho*cos(phi)*rho^2*sin(phi),phi=0..Pi/4),rho=0..a
  ),theta=0..2*Pi);
```

$$Mxy := \frac{a^4\pi}{8}$$

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> zbar:=Mxy/Vol;
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$$zbar := \frac{3a}{16\left(-\frac{\sqrt{2}}{2}+1\right)}$$

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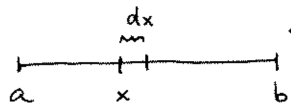
>
```

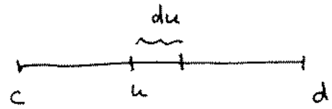
Big picture

1210:



$x = g(u)$

(in this picture
$g(c) = a$
$g(d) = b$;
g increasing)



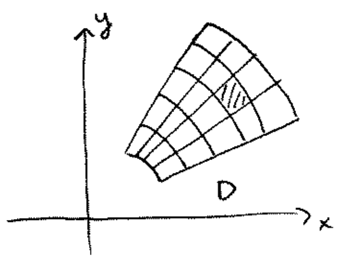
partition $[c,d]$, use g to get partition of $[a,b]$

$\Delta x_i \approx g'(u)\,\Delta u_i$

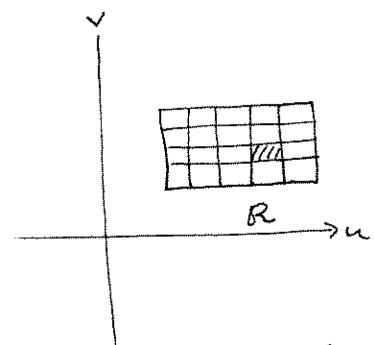
$dx = g'(u)\,du$

$$\boxed{\int_a^b f(x)\,dx = \int_c^d f(\underbrace{g(u)}_{x})\,\underbrace{g'(u)\,du}_{dx}}$$

2210: Double integrals



$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x(u,v) \\ y(u,v) \end{bmatrix} = \vec{g}(u,v)$$

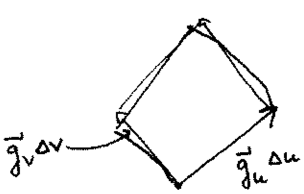


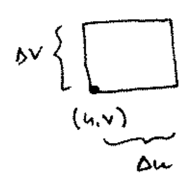
$$\iint_D f(x,y)\,dA$$

partition D
by partitioning R
and applying $\vec{g}(u,v)$.

e.g. polar coord
c.o.v.
$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} r\cos\theta \\ r\sin\theta \end{bmatrix}$

magnify shaded pieces:





$$\Delta A \approx \text{abs}\left| \vec{g}_u \Delta u \,\vdots\, \vec{g}_v \Delta v \right|$$

$$= \text{abs}\begin{vmatrix} x_u \Delta u & x_v \Delta v \\ y_u \Delta u & y_v \Delta v \end{vmatrix} = \underbrace{\text{abs}\begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}}_{\left|\frac{\partial(x,y)}{\partial(u,v)}\right|} \Delta u \Delta v$$

"Jacobian"

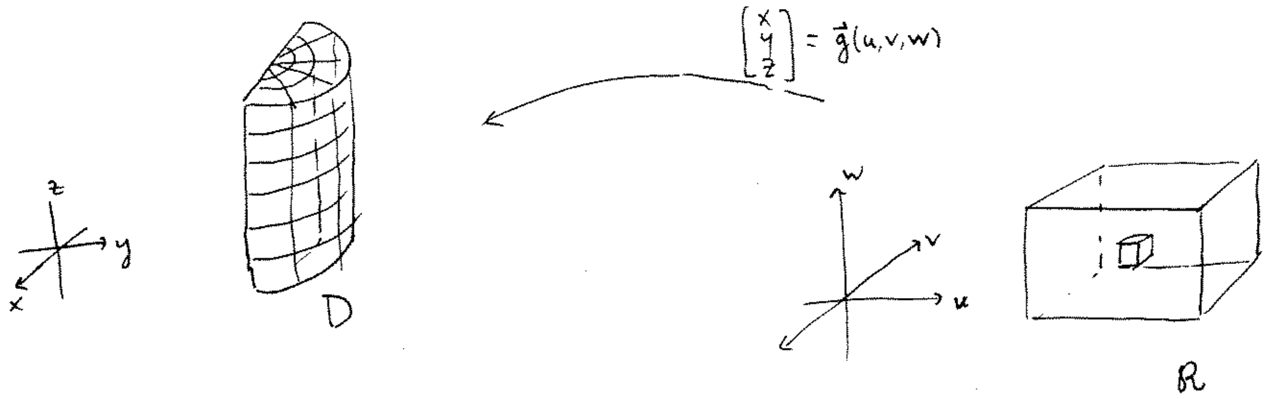
$$\boxed{dA = \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv}$$

$$\boxed{\iint_D f(x,y) = \iint_R f(x(u,v), y(u,v)) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv}$$

<u>Example</u> : check this with polar coords $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} r\cos\theta \\ r\sin\theta \end{bmatrix}$

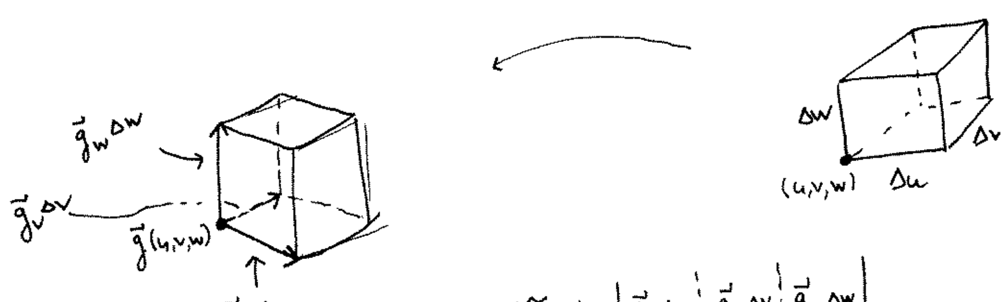
Do you get $\boxed{dA = r\,dr\,d\theta}$?

triple integrals :



$$\iiint_D f(x,y,z)\,dV$$

magnify!



$$\Delta V \cong \text{abs}\left| \vec{g}_u \Delta u \;\vdots\; \vec{g}_v \Delta v \;\vdots\; \vec{g}_w \Delta w \right|$$

$$\boxed{dV = \text{abs}\begin{vmatrix} x_u & x_v & x_w \\ y_u & y_v & y_w \\ z_u & z_v & z_w \end{vmatrix} du\,dv\,dw}$$

$$\boxed{\iiint_D f(x,y,z)\,dV = \iiint_R f(\vec{g}(u,v,w)) \left| \frac{\partial(x,y,z)}{\partial(u,v,w)} \right| du\,dv\,dw}$$

in HW you check this agrees for c.o.v. to spherical coords!