MATHEMATICS 2210-4
Calculus III, Multivariable Calculus
Spring semester 2005

text: *Calculus, 8th edition*
by Varberg, Purcell, Rigdon
when: MWF 11:50-12:40
where: LCB 219

instructor: Prof. Nick Korevaar
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office hours (may change): M, W, F 1-1:50, T 11-12:30, Th 11:50-12:40 and by appointment. Note: Thursday hour is problem session, room to be announced

2210-1 home page: www.math.utah.edu/ ~korevaar/2210spring05

prerequisites: Math 1210 and 1220 or equivalent (for example, an AP score of at least 3 on the BC Calculus exam). For our Department's placement recommendations, visit www.math.utah.edu/ugrad/ap.html.

course outline: This is the final course in the three-semester Calculus sequence, Mathematics 1210-1220-2210, and covers chapters 13-17 of the text. As you have already been learning, Calculus is part of the mathematical foundation with which science can model the world. Isaac Newton (1642-1727) was one of its co-discoverers, and his aim was to understand the physics he saw in the natural world, such as planetary motion. A beautiful quote of Galileo, from 1623, anticipates the mathematics which has followed:

> *Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it.*

In single-variable Calculus the domain and range (input and output) of the functions you consider are both subsets of the real numbers. In real-world problems it is often more natural to consider multivariable possibilities, for the domain, the range, or both. Thus, if you want to model planetary (or other) motion, the input can be the real variable t, for time, but the output position function should either describe points in space (i.e. 3-dimensional real space, $\mathbb{R}^3$), or in a plane $\mathbb{R}^2$ containing the sun and the planet. If you want to study the temperature in Utah, the input could be described by (x, y, t), where (x, y) are used to describe where you are in Utah, and t is time, and the output temperature $T(x, y, t)$ is a real number. If you want to describe electrical or magnetic fields, or complete weather systems, or any complicated system in science, engineering, business, medicine or industry, then the inputs and ouputs are usually both multivariable. Our goal in Math 2210 is to adapt and extend the ideas of the derivative, the integral, and the Fundamental Theorem of Calculus to multivariable settings, and to study important applications which result.

It is a good idea to understand the geometry and algebra of the plane $\mathbb{R}^2$, 3-space $\mathbb{R}^3$, and n-space $\mathbb{R}^n$, in order to understand functions between them, and so we do this at the start of the course, beginning with Chapters 13-14 of the text, and a study of vectors and vector operations. These chapters also develop Calculus for functions with real-number input and multivariable ($\mathbb{R}^n$) output, such as those which describe particle motion. This is an important context to consider, and one you will return to in physics, as well as in Math 2250 or 2280, when you consider systems of ordinary differential equations. The primary calculus objects we consider will be the tangent (velocity) and acceleration vectors associated to particle motion, as well as something called the "curvature" which measures how much the image curve (traced out by the particle) is bending. Time permitting, we will illustrate the value of our work by giving a version of Newton's famous deduction that *only* the inverse square law of graviational acceleration is consistent with Kepler's observed laws of planetary motion - one of the great breakthroughs in the history of science.

Chapter 15, *The Derivative in n-Space*, is a study of differential calculus for real-valued functions when the domain is multivariable, the reverse of the situation in chapters 13-14. The notion of differentiability for such functions has a new twist based on linear approximation. We will meet new versions of the chain rule, critical points for optimization problems, and the second derivative test. Many of you will study functions with multivariable domains in courses about partial differential equations, for example in Math 3150, or in physics courses about electricity and magnetism.

Chapter 16, *The Integral in n-Space*, is a study of definite integration for real valued functions of several variables, i.e. when the domain is a subset of the plane or 3-space rather than just an interval in the real numbers. It will turn out that we can reduce these integral problems to iterated 1-variable integrals, at which you are already experts. Understanding how to change variables in multiple integrals will be interesting. Applications of double and triple integrals include the computation of areas and surface areas, volumes, masses and moments.

Chapter 17, *Vector Calculus*, is analagous to the Fundamental Theorem of Calculus and its applications in 1210-1220. There are various versions of these generalizations, and they all relate integrals of certain (partial) derivatives of functions over domains in $\mathbb{R}^n$, to boundary integrals. Included in this zoo of results are Green's, Gauss', and Stoke's Theorems, which are the foundation of classical physics topics such as electro-magnetism and fluid mechanics.

grading: There will be two midterms, a comprehensive final examination, and homework. Each midterm will count for 20% of your grade, homework will count for 30%, and the final exam will make up the remaining 30%.

Homework will be assigned daily and collected weekly, on Fridays. Our grader will grade a subset of the problems you hand in. You are strongly encouraged to collaborate as necessary on homework problems, and to use whatever technology you find helpful. You must each complete and hand in your own problem set, however, and you will not learn the material if you just copy someone else's solutions. I use homework problems to let you drill basic skills, but also to explore more in-depth problems and applications. The value of carefully working homework problems is that mathematics (like anything) must be practiced and experienced to be learned, so make sure that you really understand each problem you hand in.

I will arrange and announce a classroom for our optional Thursday problem sessions, at this class time 11:50-12:40, and you are all invited. I know from previous experience that some students flourish without attending these sessions, but that others find them quite helpful; if you think you fall in the latter group you may wish to arrange your schedule so that you can attend the problem sessions. In addition to this special problem session, the Math Department Tutoring Center is located in Rushing Student Center between LCB and JWB. It is open for free tutoring from 8 a.m. to 8 p.m. on M-Th, and from 8 a.m. to 6 p.m. on Friday. (But often the tutoring center doesn't open until several days into the semester.) Some, but not all of the math tutors welcome questions from Math 2210 students. To see the times and specialities of various tutors, consult the web address www.math.utah.edu/ugrad/tutoring.html .

It is the Math Department policy, and mine as well, to grant any withdrawl request until the University deadline of Friday March 4.

ADA statement: The American with Disabilities Act requires that reasonable accomodations be provided for students with physical, sensory, cognitive, systemic, learning, and psychiatric disabilities. Please contact me at the beginning of the semester to discuss any such accommodations for the course.

Tentative Daily Schedule

exam dates fixed,
daily subject matter approximated

M	10 Jan	14.1	$3-$space ($\mathbb{R}^3$), distance
W	12 Jan	13.2,13.3,14.2	vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$
F	14 Jan	13.2,13.3,14.2	dot product algebra and geometry
M	17 Jan	none	Martin Luther King Day
W	19 Jan	14.3	cross product algebra and geometry
F	21 Jan	14.4, 14.6	lines and planes in space
M	24 Jan	14.6	cylinders and quadric surfaces
W	26 Jan	14.7	cylindrical and spherical coordinates
F	28 Jan	13.1, 14.4	parametric curves
M	31 Jan	13.4,14.5	velocity, acceleration, arclength, curvature
W	2 Feb	13.4,14.5	continued
F	4 Feb	extra	Kepler and Newton
M	7 Feb	15.1	functions of several variables
W	9 Feb	15.3	limits and continuity
F	11 Feb	15.2	partial derivatives
M	14 Feb	review	chapters 13-15.3
W	16 Feb	Exam 1	chapters 13-15.3
F	18 Feb	15.7, 15.4	tangent planes, linear approximation, differentiability
M	21 Feb	none	Presidents' Day
W	23 Feb	15.6	chain rule
F	25 Feb	15.5	directional derivatives
M	28 Feb	15.8	max-min problems
W	2 Mar	15.9	Lagrange multipliers
F	4 Mar	16.1	double integrals over rectangles
M	7 Mar	16.2	iterated integrals
W	9 Mar	16.3	double integrals over nonrectangular domains
F	11 Mar	16.4	double integrals in polar coordinates
M	14 Mar	none	spring break!
W	16 Mar	none	spring break!
F	18 Mar	none	spring break!

M	21 Mar	16.5	applications of double integrals
W	23 Mar	16.6	surface area
F	25 Mar	16.7	triple integrals
M	28 Mar	16.8	integrals in cylindrical and spherical coordinates
W	30 Mar	extra	change of variables in multiple integrals
F	1 Apr	review	15.4-16.8 + extra
M	4 Apr	exam 2	15.4-16.8 + extra
W	6 Apr	17.1	vector fields
F	8 Apr	17.2	line integrals
M	11 Apr	17.3	independence of path
W	13 Apr	17.4	FTC and Green's Theorem in the plane
F	15 Apr	extra	divergence and curl versions of Green's Theorem
M	18 Apr	extra	parametric surfaces and their area
W	20 Apr	17.5 + extra	surface integrals
F	22 Apr	17.6	Gauss' divergence theorem
M	25 Apr	17.7	Stokes' curl theorem
W	27 Apr	13-17	review
F	29 Apr	none	University reading day
M	2 May	FINAL EXAM	entire course, 10:30-12:30 a.m., in classroom