Math 2210-1
Exam Wed!

today 12.3: quadric surfaces
(see also, Maple handout)

quadric surfaces are solution sets to a single quadratic eqtn in 3 variables

$$ax^2 + by^2 + cz^2$$
$$+ dxy + exz + fyz$$
$$+ gx + hy + jz + k = 0.$$

So they are the $\mathbb{R}^3$ analog of conic sections in $\mathbb{R}^2$.
We'll assume no cross terms $d = e = f = 0$,
and usually (at least in class), that we've completed the square and translated our axes, so that $g = h = j = 0$, whenever the corresponding a, b, c are $\neq 0$.

To draw (and visualize) these surfaces it helps to draw their intersections ("traces") with various planes // to the coord planes.

<u>examples</u>: "cylinders": at least one of the variables does not appear in the eqtn.

1 <u>example</u> $\cancel{x}z^2 + \frac{y^2}{4} = 1$

this is a cylinder in the x-direction, since if (x_1, y, z) is on the surface, so is (x_2, y, z).

— So if you draw the traces on 2 planes $\perp$ to the x-axis, you can draw the cylinder.

(2) ellipsoids

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

e.g. $\frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1$

→ draw the trace ellipses
in the coord planes.

(3) paraboloids: one variable only appears linearly (and the others appear quadratically)

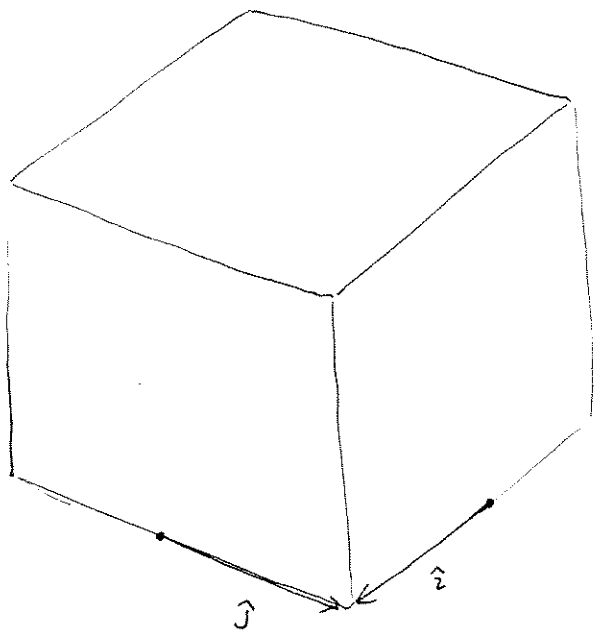
elliptic paraboloid : quadratic terms have same sign

e.g. $z = x^2 + y^2$

→ use the trace parabolas
is xz, yz planes,
and the trace ellipses
in planes ⊥ to z-axis.

<u>hyperbolic paraboloid</u> : quadratic terms have opposite sign

e.g. $z = x^2 - y^2$



origin at center of box.
sides are planes at

$x = \pm 1$
$y = \pm 1$
$z = \pm 1$.

draw trace parabolas!

4 hyperboloids

4a) one-sheeted, e.g.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

e.g. $x^2 + y^2 - z^2 = 1$

or $x^2 + y^2 = 1 + z^2$

→ use traces (ellipses) in planes ⊥ to z axis, and in coord planes

4b) two-sheeted. e.g.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$$

e.g.

$x^2 + y^2 - z^2 = -1$

or $x^2 + y^2 = z^2 - 1$

4c) cones, e.g.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$$ ~ if (x_1,y_1,z_1) is on the surface, so is the line $\vec{r}(t)=t\langle x_1,y_1,z_1\rangle$

e.g. $x^2+y^2-z^2=0$