Math 2210
Wed 9/19

- Finish matrix algebra discussion, page 4-5 Monday notes

- Inverse matrices

HW for Wed 9/22

11.6 (32, 41) 53, 54, 55

12.1 1, 3, 6 ← & draw a picture with curve, $\vec{r}(2)$, $\vec{r}'(2)$, $\vec{r}''(2)$

8, 15, 18, 33, 35, 36, 39, 43, 45, 46, 55, 60

12.2 1, 5, 7, 8, 16, 17

Schedule (?):

today	11.6
F	12.1
M	12.2
W	12.3
F	12.4
M	Kepler (from 12.2)
W	exam 1, chapters 11 & 12

- $n \times n$ identity matrix

$$I = \begin{bmatrix} 1 & 0 & 0 & & 0 \\ 0 & 1 & 0 & & \vdots \\ \vdots & 0 & 1 & & \\ 0 & \vdots & 0 & & 0 \\ & & 0 & 0 & 1 \end{bmatrix}$$

$$I_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

$I\vec{x} = \vec{x}$ for all n-vectors $\vec{x}$

- Let $A_{n\times n}$ correspond to $\vec{F}(\vec{x}) = A\vec{x}$
$\vec{F} : \mathbb{R}^n \to \mathbb{R}^n$

If $\text{rref}(A) = I$ then $A\vec{x} = \vec{b}$ always has a unique solution (why?)

So $\vec{F}$ has inverse function.

We can find its matrix B (which must satisfy

$AB = I$, $BA = I$

$(F(F^{-1}(\vec{x})) = \vec{x})$ $(F^{-1}(F(\vec{x})) = \vec{x})$

called A^{-1}

By solving

$A X_{n\times n} = I$:

<u>example</u> Find A^{-1} for $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right]$$

← stands for $A \cdot col_1(A^{-1}) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$A \cdot col_2(A^{-1}) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

$$\begin{array}{r} \\ -R_1+R_2 \end{array} \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{array}\right]$$

$$\begin{array}{r} -R_2+R_1 \\ \\ \end{array} \left[\begin{array}{cc|cc} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 1 \end{array}\right]$$

↑ for $col_1(A^{-1})$
$x = 2$
$y = -1$

← for $col_2(A^{-1})$
$x = 1$
$y = 1$

so $A^{-1} = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}$!

check this!

If you know A^{-1}, then to solve

$$A\vec{x} = \vec{b}$$

$$A^{-1}(A\vec{x}) = A^{-1}\vec{b}$$

$$(A^{-1}A)\vec{x} = A^{-1}\vec{b}$$

$$I\ \vec{x} = A^{-1}\vec{b}$$

$$\vec{x} = A^{-1}\vec{b}$$

or, in function language, if $\vec{F}^{-1}$ exists and

$$F(\vec{x}) = \vec{b}$$

then $\vec{x} = \vec{F}^{-1}(\vec{b})$

<u>example</u>: Solve $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ using A^{-1}

<u>example</u>: Find A^{-1} for $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

ans = $\begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & -4 \\ 1 & -1 & 1 \end{bmatrix}$.