# MAPLE for Math 2210

Friday Sept 10, 2004

You may use any technology you want to complete or check your homework solutions in our section, Math 2210-1. Sometimes the point of the problem is to let you practice by hand, in which case you should do the computation by hand, but feel free to check it with technology. Other times, the point of the problem is the mathematical concept and you can let your graphing calculator or the computer do the grungy details. An especially good example of this is the homework problem from last week in which you were to find the area of a messy quadrilateral - if you look at the posted solutions, I set up the problem and then had the computer do the messy math.

This handout is being made using a package called MAPLE, which exists on our Math system (to which you all have accounts), as well as at Marriott and Engineering. (You can also buy a version from the University bookstore for about $120.) I will be happy to help any students interested in using this software -which as you can see allows you to mix word processing with mathematical computations.

Today we'll use MAPLE to do work and illustrate concepts related to sections 11.4-11.6 of our text. A typical sort of problem would be like %11.4 #15 from last week, in which you were to find whether two lines intersected, and if so where. If you understand how to set this problem up, it is easy to have the computer find the answer. Here are four ways to attack the same problem, illustrating some of the concepts we've been talking about:

**Problem:** *Let two lines be given in symmetric form by*

$$L_1:\ x-2 = \frac{y}{2}+\frac{1}{2} = \frac{z}{3}-1$$

$$L_2:\ \frac{x}{3}-\frac{5}{3} = \frac{y}{2}-\frac{1}{2} = z-4$$

*Determine whether lines intersect, are skew, or parallel.*

First, note that the direction vector of the first line is <1,2,3>, and for the second line it's <3,2,1>. Since these vectors are not parallel (i.e. multiples of each other), the lines either intersect or are skew.

**Method 1)** "**solve**" command": we are really asking for whether four planes intersect, since the symmetric form of a line can be interpreted as giving the line as an intersection of two planes.

```
> solve({x-2=(y+1)/2,x-2=(z-3)/3,
        (x-5)/3=(y-1)/2,(x-5)/3=z-4},{x,y,z});
```

$$\{y=-1, z=3, x=2\}$$

**Method 2)** Write the 4 linear equations above as a system, do elementary row operations to find the solution. We can write the
equations as

$$2x-y=5$$

$$3x-z=3$$

$$2x-3y=7$$

$$x-3z=-7$$

Which, when we write it synthetically gives us the *augmented* matrix A (The matrix of coefficients is augmented with the column of right hand sides, hence the name *augmented*):

```
> with(linalg):  #load matrix and linear algebra package
  A:=matrix(4,4,[2,-1,0,5,3,0,-1,3,2,-3,0,7,1,0,-3,-7]);
  rref(A);  #compute the reduced row echelon form of the matrix
Warning, the protected names norm and trace have been redefined and unprotected
```

$$A := \begin{bmatrix} 2 & -1 & 0 & 5 \\ 3 & 0 & -1 & 3 \\ 2 & -3 & 0 & 7 \\ 1 & 0 & -3 & -7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

We can read off the solution x=2, y=-1, z=3 from the rref(A).

**Method 3)** Convert the symmetric forms of the lines into a parametric equations, and use "solve" again. For example,

```
> r1:=t-><2,-1,3>+t*<1,2,3>;
  r2:=s-><5,1,4>+s*<3,2,1>;
```

$$r1 := t \to \langle 2, -1, 3\rangle + t\,\langle 1, 2, 3\rangle$$
$$r2 := s \to \langle 5, 1, 4\rangle + s\,\langle 3, 2, 1\rangle$$

```
> solve({t+2=3*s+5,2*t-1=2*s+1,3*t+3=s+4},{s,t});
```

$$\{t=0, s=-1\}$$

```
> r1(0)=r2(-1);  #get point and check answer
```

$$\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

**Method 4)** Like method (2), only to get s and t:

```
> B:=matrix(3,3,[-3,1,3,-2,2,2,-1,3,1]);
  rref(B);
```

$$B := \begin{bmatrix} -3 & 1 & 3 \\ -2 & 2 & 2 \\ -1 & 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

i.e. s=-1 and t=0, from which you recover the point of intersection.

**Example:** Planes, parametrically: Consider the plane given implicitly by

$$x-2\,y+z=4$$

This single equation corresponds to a very short augmented matrix

```
> C:=matrix(1,4,[1,-2,1,4]);
```

$$C:=[1 \quad -2 \quad 1 \quad 4]$$

This matrix is already in reduced row echelon form, and we can backsolve: z=t, y=s, x=4+2*s-t. We can write this in vector form:

```
> F:=(s,t)-><4,0,0>+s*<2,1,0>+t*<-1,0,1>; #F is the position vector
      #of points on the plane
```

$$F:=(s,t)\rightarrow\langle 4,0,0\rangle+s\,\langle 2,1,0\rangle+t\,\langle -1,0,1\rangle$$

Can you explain, with your understanding of what vector addition and scalar multiplication, why this collection of points gives a plane? We can check with a plotting command:

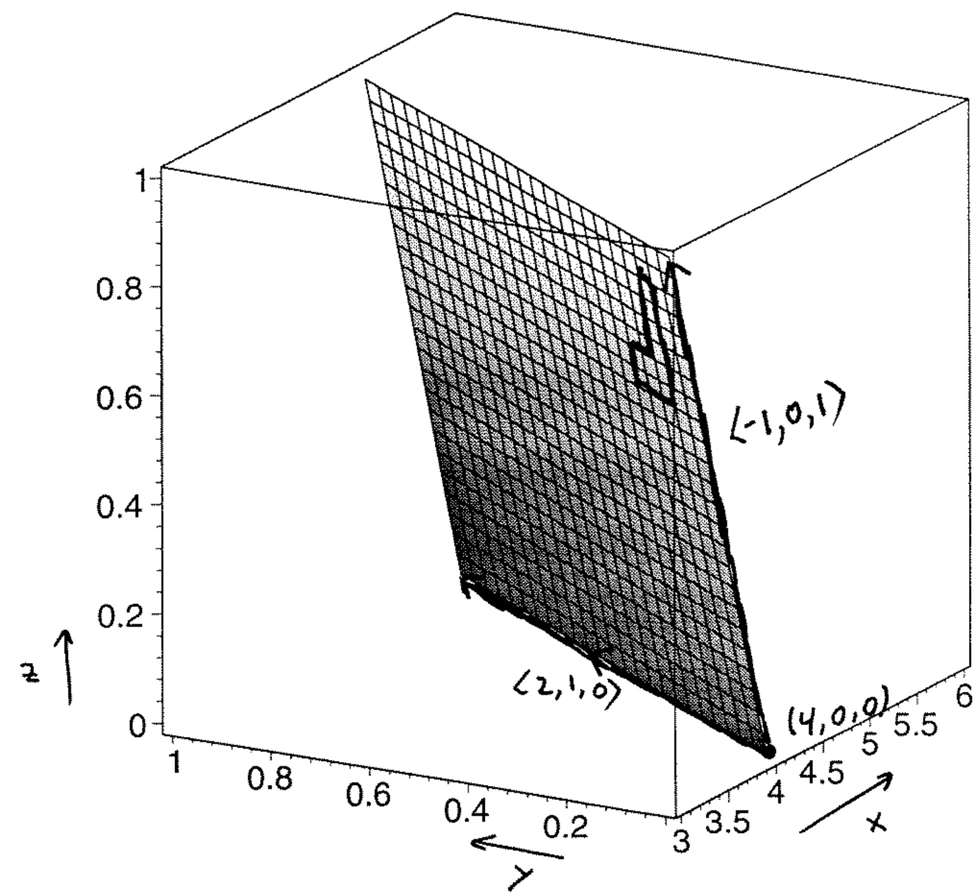
```
> with(plots);  #load the plotting package

Warning, the name changecoords has been redefined
```

[*animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, cylinderplot, densityplot, display, display3d, fieldplot, fieldplot3d, gradplot, gradplot3d, graphplot3d, implicitplot, implicitplot3d, inequal, interactive, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, replot, rootlocus, semilogplot, setoptions, setoptions3d, spacecurve, sparsematrixplot, sphereplot, surfdata, textplot, textplot3d, tubeplot*]

```
> ?plot3d;
> plot3d([4+2*s-t,s,t],s=0..1,t=0..1,axes=boxed);
```

$\vec{F}(s,t) = \langle 4,0,0\rangle + s\langle 2,1,0\rangle + t\langle -1,0,1\rangle$




Would you like to try one of your homework problems from 11.5?

What does all this have to do with multivariable Calculus, i.e. Math 2210??

Well, think about Calculus...

The first fcns you studied were "affine" fcns (you might have called them "linear" because their graphs were lines, but affine is the correct term)

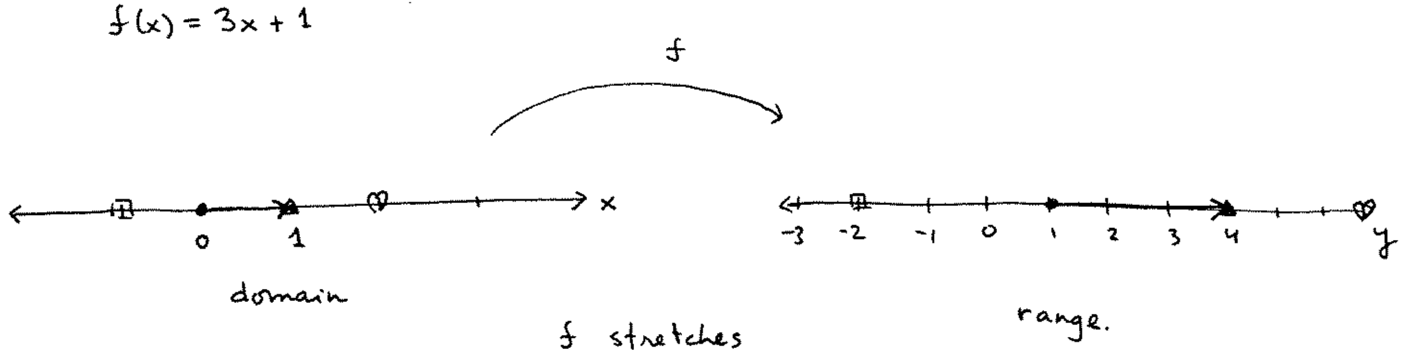
$$f(x) = mx + b$$

1210: $m$ = slope
$b$ = y-intercept.

2210: $m$ = scale or stretch factor
$b$ = translation

e.g.

$$f(x) = 3x + 1$$

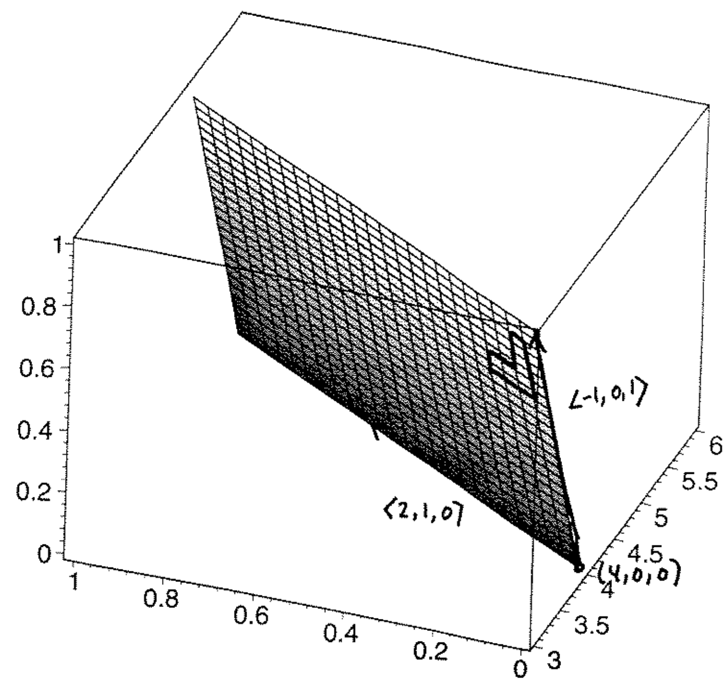



f stretches (scales) by a factor of 3 and then translates by +1.

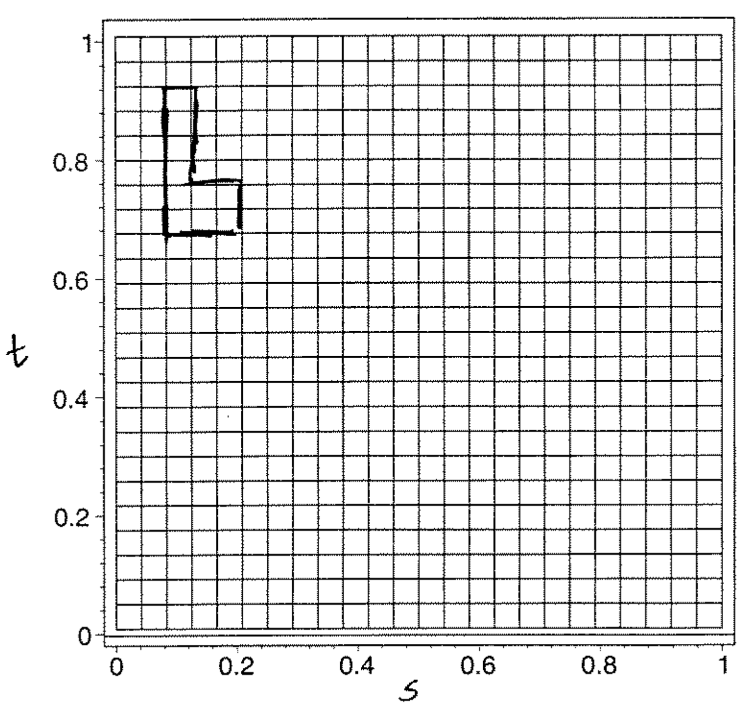
You may not have realized it, but the "moral" of Calculus is that differentiable functions are essentially affine functions when you look at small scales. For example, here is an "explanation" of the chain rule:

If $f(x) = mx + b$
& $g(y) = My + B$
then $g(f(x)) = M(mx+b) + B = Mmx + C$
so $\frac{d}{dx} g(f(x)) = Mm = g'(f(x)) \cdot f'(x)$

Other Calculus ideas have affine approximation at their heart
Multivariable calculus is built on the "same" foundation...




$$\vec{F}\left(\begin{bmatrix}s\\t\end{bmatrix}\right) = s\begin{bmatrix}2\\1\\0\end{bmatrix} + t\begin{bmatrix}-1\\0\\1\end{bmatrix} + \begin{bmatrix}4\\0\\0\end{bmatrix} = \begin{bmatrix}2s-t\\s\\t\end{bmatrix} + \begin{bmatrix}4\\0\\0\end{bmatrix} = \begin{bmatrix}2&-1\\1&0\\0&1\end{bmatrix}\begin{bmatrix}s\\t\end{bmatrix} + \begin{bmatrix}4\\0\\0\end{bmatrix}$$



Multivariable affine transformations

$\vec{F}: \mathbb{R}^n \to \mathbb{R}^m$

$\vec{F}(\vec{x}) = A\vec{x} + \vec{b}$

$$\underset{\text{m-vector}}{\vec{F}\left(\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}\right)} = \underset{\substack{m\times n \\ \text{matrix}}}{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & & & \\ \vdots & & & \\ a_{m1} & & \cdots & a_{mn} \end{bmatrix}} \underset{\substack{n \\ \text{vector}}}{\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}} + \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + b_1 \\ a_{21}x_1 + \cdots + b_2 \\ \vdots \end{bmatrix} = x_1 \overset{\substack{\text{1st col} \\ \downarrow}}{\begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}} + x_2 \overset{\substack{\text{2nd col} \\ \downarrow}}{\begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \end{bmatrix}} + \cdots + x_n \begin{bmatrix} a_{1n} \\ \vdots \\ a_{mn} \end{bmatrix} + \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}$$

$$= \begin{bmatrix} \text{Row}_1(A)\cdot\vec{x} \\ \text{Row}_2(A)\cdot\vec{x} \\ \vdots \\ \text{Row}_m(A)\cdot\vec{x} \end{bmatrix} + \vec{b}$$

To understand the geometry of general affine maps we'll focus on $\vec{F}: \mathbb{R}^2 \to \mathbb{R}^2$, see e.g. HW.

HW due Wed 9/15 changed!

11.5 1, 11, 15, 17, 22, 28, 29, 31 33

2 class exercises page 6

11.6 1, 10, 16 20, 32, 41 54, 55

if we get this far on Monday!