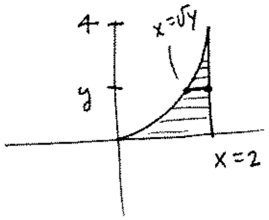
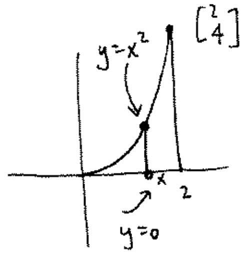
Math 2210-1
Practice Exam Solutions

1a) $\int_1^2\left(\int_0^2 xy\,dy\right)dx = \int_1^2 \underbrace{\frac{xy^2}{2}\Big]_0^2}_{2x}\,dx = x^2\Big]_1^2 = 4-1 = \boxed{3}$

b) $y = 0..4$
for each y, $\underbrace{x=\sqrt{y}}_{\text{i.e. } y=x^2}\,..\,2$

so, reversing order, $x = 0..2$
for each x, $y = 0..x^2$



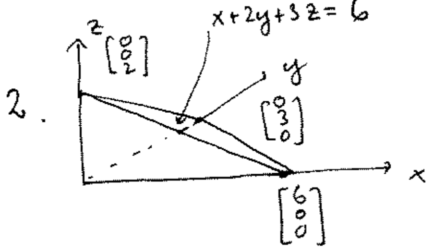


so

$$\int_0^4\int_{\sqrt y}^2 \frac{ye^{x^2}}{x^3}\,dx\,dy = \int_0^2\left(\underbrace{\int_0^{x^2}\frac{ye^{x^2}}{x^3}\,dy}_{\frac{e^{x^2}}{x^3}\underbrace{\int_0^{x^2} y\,dy}_{\underbrace{\frac{y^2}{2}\Big]_{y=0}^{x^2}}_{\frac{x^4}{2}}}}\right)dx$$

$$\int_0^2 \frac{x^4}{2}\frac{e^{x^2}}{x^3}\,dx = \frac12\int_0^2 xe^{x^2}\,dx = \frac12\left[\frac12 e^{x^2}\right]_0^2 = \boxed{\frac14\left(e^4-1\right)}$$

2.

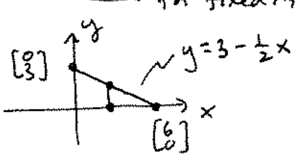


2a) as a check, this is a "cone",
so Vol $= \frac13 \underset{\substack{\uparrow\\ \text{base area}}}{A}h = \frac13\left(\frac12 6\cdot 3\right)\cdot 2 = 6$

for fixed x, y,
$0 \le z \le \frac13(6-x-2y)$

$$\int_0^6\left(\iint\left(\int_0 1\,dz\right)\right)dx$$

for fixed x, $0 \le y \le 3-\frac12 x$



$$\int_0^6\left(\int_0^{3-\frac12 x}\left(\underbrace{\int_0^{\frac13(6-x-2y)} 1\,dz}\right)dy\right)dx$$

$\frac13(6-x-2y)\,dy$

$2y-\frac13 xy-\frac{y^2}{3}\Big]_0^{3-\frac12 x}$

$$\int_0^6 \underbrace{2\left(3-\tfrac12 x\right)-\tfrac13 x\left(3-\tfrac12 x\right)-\frac{\left(3-\frac12 x\right)^2}{3}}\,dx$$

$\left.\begin{array}{l} x^2\left(\frac16-\frac1{12}\right)\\ +\ x\,(-1-1+1)\\ +\ 1\,(6-3)\end{array}\right\}$ $\frac1{12}x^2-x+3$

$\frac1{36}x^3-\frac{x^2}{2}+3x\Big]_0^6 = 6-18+18 = \boxed{6}$

2b) $\bar{x} = \dfrac{\iiint x\, dV}{V}$ (assuming $\delta = 1$)

numerator: $\displaystyle\int_0^6 \int_0^{3-\frac{1}{2}x} \int_0^{\frac{1}{3}(6-x-2y)} x \, dz\, dy\, dx$

x is const w.r.t. y, z, so can use previous page:

$$= \int_0^6 x \left[\underbrace{\int_0^{3-\frac{1}{2}x} \int_0^{\frac{1}{3}(6-x-2y)} dz\, dy}_{\frac{1}{12}x^2 - x + 3 \text{ by (2a)}} \right] dx$$

$$= \int_0^6 \frac{1}{12}x^3 - x^2 + 3x \, dx$$

$$= \frac{1}{48}x^4 - \frac{x^3}{3} + \frac{3}{2}x^2 \Big]_0^6 = \frac{\cancel{6^4}^{\,6^3}}{\cancel{6}\cdot 8} - \frac{6^3}{3} + \frac{3 \cdot 6^2}{2}$$

$$= 6^2 \left[\underbrace{\frac{3}{4} - 2 + \frac{3}{2}}_{\underbrace{\frac{3-8+6}{4}}_{\frac{1}{4}}} \right] = \frac{36}{4} = 9.$$

$\boxed{\text{So } \bar{x} = \frac{9}{6} = 1.5}$

3. $\displaystyle\iint_{D} \frac{1}{1+x^2+y^2}\, dA$. Use polar coords

$$= \int_0^{2\pi} \int_0^2 \underbrace{\frac{1}{1+r^2} r\, dr}_{\frac{1}{2}\ln(1+r^2)\Big]_0^2} d\theta$$

$$\int_0^{2\pi} \frac{1}{2}\ln 5 \, d\theta = 2\pi \cdot \frac{1}{2}\ln 5 = \boxed{\pi \ln 5}$$