Name		 	
I.D. number	••••••	 	

Math 2210-1 Practice Exam 1 September 27, 2004

This exam is closed-book and closed-note. You may use a scientific calculator, but not one which is capable of graphing or of solving linear algebra equations. In order to receive full or partial credit on any problem, you must show all of your work and justify your conclusions. There are 100 points possible. The point values for each problem are indicated in the right-hand margin. (On the actual exam there will be space for you to do your work. On this practice exam use your own paper.) Good Luck!

1a) Consider the triangle with vertices the origin 0=(0,0,0), and the points P=(1,2,1) and Q=(0,1,-1). Find the area of this triangle.

- (10 points) 1b) Find an (implicit) equation of the form ax+by+cz=d for the plane containing the three points in (1a). (5 points)
- 1c) Express this same plane parametrically.
- 1d) Consider the line through R=(1,1,0) and S=(0,1,1). Express this line parametrically.
- 1e) Find the intersection of the line in (1d) with the plane in (1a).
- 1f) Consider a second plane, given implicitly by

$$x - y - z = 2$$

Find the intersection set of this plane with the plane in (1a).

(10 points)

(5 points)

(5 points)

(10 points)

2) Consider an affine map from R^2 to R^2 . Consider the following L-box diagram, which shows the unit "L-box" and where it is transformed to by our mystery affine map:



2a) Find the formula for the mystery affine map.

(15 points)

2b) What is the area of the image L-box?

(5 points)

3) Consider the parametric curve given by

$$\mathbf{r}(\mathbf{t}) = \begin{bmatrix} t^2 - t \\ t^2 + 1 \end{bmatrix}$$

Here is a sketch of the range of **r**, for t between zero and 1.5:



3a) Compute $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$.

(4 points) 3b) Write down a definite integral which yields the length of the parametric curve $\mathbf{r}(t)$, for t in the interval between zero and 2. Do not evaluate this integral. (6 points) 3c) Label the point with position vector $\mathbf{r}(1)$ into the picture above. Add the vectors $\mathbf{r}'(1)$ and $\mathbf{r}''(1)$ to the picture, in the appropriate locations. (5 points) 3d) Find the component of $\mathbf{r}''(1)$ in the direction of $\mathbf{r}'(1)$ using the dot product. (5 points) 3e) In class we derived the deomposition formula for acceleration, decomposing it into tangential and normal parts. The formula was

$$\mathbf{r}$$
''(t) = $\left(\frac{d}{dt}\mathbf{v}(t)\right)T + \kappa v^2 N$

where *T* and *N* are the unit tangent and normal vectors, v is the speed, and κ is the curvature. Add the vectors *T* and *N* to your picture above. (Hint: you can find *N* easily because it is perpendicular to *T* and the curve lies in a plane.)

(4 points)

3f) Verify that the component you computed in (3d) agrees with $\frac{d}{dt}v(t)$, when t=1.

3g) Use the formula from (3e), and your previous computations, to deduce the curvature of the parametric curve $\mathbf{r}(t)$, when t=1.

(6 points)

(5 points)