Math 2210-1
Monday 10/25

on HW for Wed,
13.9 #17 not required
(2 constraints).

Exam 2 Wednesday!
12.3 – 13.9 Review Sheet:

12.3 : • identify & sketch quadric surfaces, making use of their trace curves in various planes. (Since these are conic sections you should know conics as well.
• complete the square if necessary, for translated copies

12.4 • polar, cylindrical, spherical coords

$\begin{bmatrix} r \\ \theta \end{bmatrix}$ $\begin{bmatrix} r \\ \theta \\ z \end{bmatrix}$ $\begin{bmatrix} \rho \\ \phi \\ \theta \end{bmatrix}$

know how to convert between these coord systems and rectangular coords, know geometric meaning of these coords.

13.2 • describing functions of several variables, i.e. $f: \underset{\mathbb{R}^n}{\overset{\cap}{D}} \to \mathbb{R}$

graph $z = f(\vec{x}) \rightarrow$ in $\mathbb{R}^{n+1}$
level curve $\rightarrow$ in $D \subset \mathbb{R}^2$
level surface $\rightarrow$ in $D \subset \mathbb{R}^3$
contour curve. $\rightarrow$ on graph $z = f(x,y)$, in $\mathbb{R}^3$.

13.3 limits and continuity
• be able to compute limits, know def. of continuity.

13.4 • partial derivatives:
limit definition
how to compute
geometric meaning
slope
rate of change

tangent plane to graph $z = f(x,y)$.

13.5 • multivariable optimization
A set is closed if every point to which you can get arbitrarily close to from within the set, is itself in the set

If $f: D \to \mathbb{R}$ and if D is closed and bounded
then f attains its extreme values.

all pts $\vec{x} \in D$ sat. $|\vec{x}| \leq M$ some fixed M

• How to find these?
• Applied max-min (word problems)

13.6 • linear (& affine) approximations, & matrix derivatives
13.7 • chain rule

• $\Delta f \approx df = \nabla f \cdot d\vec{x}$
• $\Delta \vec{F} \approx d\vec{F} = [F'(\vec{x})]\, d\vec{x}$

} approximation problems

• def of differentiable
• Chain rule: $[(G \circ F)'(x)] = [G'(F(x))][F'(x)]$
scalar form (as in HW)
• special case $\frac{d}{dt} f(\vec{r}(t)) = \nabla f(\vec{r}(t)) \cdot \vec{r}\,'(t)$

} computational & interpretive problems

13.8 • Directional derivatives & gradient vector

$D_{\hat{u}} f(\vec{x})$ def'n
interpretation
formula for computing
interpretation of formula.
~ what the gradient tells you.

why $\nabla f(P)$ is $\perp$ to level set thru P.
tangent line to level curve
tangent plane to level surface

13.9 • Lagrange multipliers, another way of doing constrained optimization

Practice Test questions (samples; not inclusive).

① Consider $f(x,y) = x^2 - y^2$.

(a) What quadric surface is the graph $z = f(x,y)$?
(b) Sketch the level curves $f=0$, $f=1$, $f=-1$
(c) onto this picture add the curve $x^2 + y^2 = 4$.

(d) Use Lagrange multipliers to find the maximum value of $x^2 - y^2$ subject to the constraint $x^2+y^2=4$. Add to points where this occurs to your lbc, picture, and explain how you could have deduced these locations from your understanding of the level curves of f.

② (a) What is the equation for the tangent plane to $\underbrace{z = x^2 - y^2}_{= f(x,y)}$, at $x=y=1$.

(b) Use differentials to approximate $f(1.1, .9)$. Compare to exact value
(c) How does your answer to (b) compare to the height of ~~that~~ tangent plane from (a), above the point $(1.1, .9)$? Explain.

(d) Let $F(t)$ be a curve for which $\vec{r}(0) = \begin{bmatrix}1\\1\end{bmatrix}$, $\vec{r}'(0) = \begin{bmatrix}5\\4\end{bmatrix}$.

Find $\left.\frac{d}{dt} f(\vec{r}(t))\right|_{t=0}$

(e) In what direction is f increasing most rapidly at the point $\begin{bmatrix}1\\1\end{bmatrix}$?

③ Define what it means for $\vec{F}: \mathbb{R}^n \to \mathbb{R}^m$ to be

(a) continuous at $\vec{x}$
(b) differentiable at $\vec{x}$

④ Find the derivative matrix of the spherical coord map

$$\vec{F}\begin{bmatrix}\rho\\ \phi\\ \theta\end{bmatrix} = \begin{bmatrix}\rho\sin\phi\cos\theta\\ \rho\sin\phi\sin\theta\\ \rho\cos\phi\end{bmatrix} \quad \left(= \begin{bmatrix}x\\y\\z\end{bmatrix}\right).$$

We probably want to work a Lagrange multiplier problem --
e.g. 13.9 #1, 5 ? (any from 1-15 good.)