Math 2210-1
Monday 11 Oct

Fri → talked about limits & continuity.
today: start talking about differentiability

$f: D \to \mathbb{R}$, $D \subset \mathbb{R}^2$

$$\frac{\partial f}{\partial x}(x,y) = \lim_{h\to 0} \frac{f(x+h,y) - f(x,y)}{h}$$ ← hold y fixed, vary in the x-dir.

$$\frac{\partial f}{\partial y}(x,y) = \lim_{k\to 0} \frac{f(x,y+k) - f(x,y)}{k}$$ ← hold x fixed, vary in y-dir.

("partial derivative in y dir"
(measures how fast f is changing in y-direction.).

<u>examples</u>: (since only 1 variable is varying, can use 1210 to compute partial derivs!)

$f(x,y) = e^{xy}\cos y$

compute $\frac{\partial f}{\partial x} =$

$\frac{\partial f}{\partial y} =$

analogous if more variables:

$g(x,y,z) = z^2\cos(xyz)$

<u>define</u> $\frac{\partial g}{\partial z}$

compute $\frac{\partial g}{\partial z}$

rates of change interpretation:

example: The temperature (° Centigrade) of a rod of length π ($0 \le x \le \pi$) meters is given at time t minutes by

$$T(x,t) = 20(\sin x)e^{-2t}$$

What is

(a) $T(\pi/2, 0)$

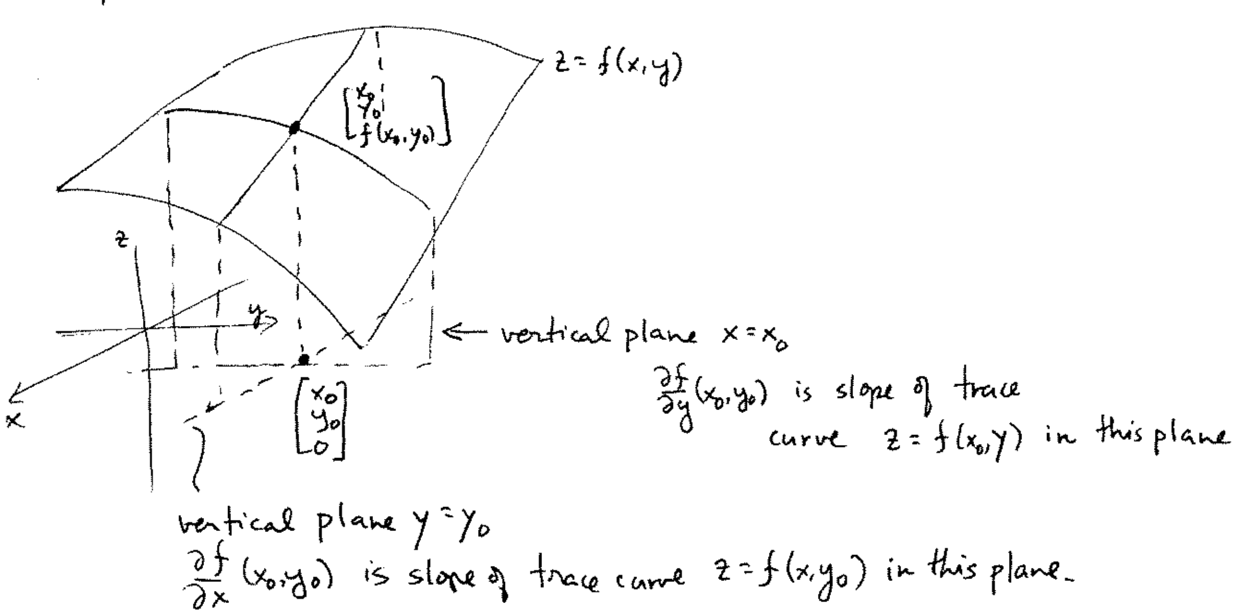
(b) how fast is temperature changing at $x = \pi/2$, when $t = 0$?

(c) how is temperature changing wrt the x-variable, at $x = \pi/2$, $t = 0$?

geometric interpretation:

If $f: D \to \mathbb{R}$, $D \subset \mathbb{R}^2$

we can interpret partial derivatives geometrically in terms of the graph $z = f(x,y)$:



Tangent plane to the graph of f:

If $f: \underset{\mathbb{R}^2}{\underset{\cap}{D}} \to \mathbb{R}$ and if the graph of f has a tangent plane at $(x_0, y_0, f(x_0, y_0))$

its eqtn will be

$$z = a(x - x_0) + b(y - y_0) + f(x_0, y_0)$$

want partial derivs to match at (x_0, y_0)

so $a = \frac{\partial f}{\partial x}(x_0, y_0)$

$b = \frac{\partial f}{\partial y}(x_0, y_0)$

<u>example</u> for $f(x,y) = x^2 + y^2$

find eqtn of tangent plane to graph of f,

at $(x,y) = (0,1)$.

sketch.

notation and higher order partial derivs:

$\frac{\partial f}{\partial x}$, f_x same

$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) := \frac{\partial^2 f}{\partial y \partial x} := (f_x)_y = f_{xy}$

$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2} = f_{xx}$

etc.

compute all 1st & 2nd order partial derivs for

$f(x) = e^{2x} \cos 3y$

What do you ~~k~~notice about f_{xy} & f_{yx}?