HW for Wed Oct 6

12.3 1, 12, 16, 23, 28, 29, 30

12.4 5, 11, 12, 18, 21, 26, 31, 33, 36, 39, 49, 50, 55

Math 2210-1
Fri 10/1

§ 12.4 Alternate coordinates in $\mathbb{R}^2$ & $\mathbb{R}^3$

$\mathbb{R}^2$: polar coordinates

(we used these for Kepler; many of you saw them at the end of 1220).

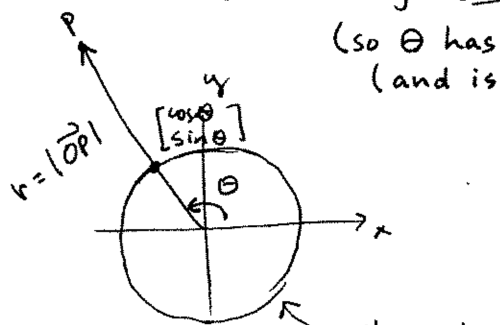
For $P=(x,y) \in \mathbb{R}^2$

$r := \sqrt{x^2+y^2} = dist(P, O)$

$\theta :=$ the angle from the x-axis to $\overrightarrow{OP}$

(so θ has a sign)

(and is uniquely defined up to a multiple of 2π



$r = \sqrt{x^2+y^2}$

$$\theta = \begin{cases} \arctan y/x & \text{quadrant I, IV} \quad (-\pi/2 \le \theta \le \pi/2) \\ \arccos \frac{x}{r} & \text{I, II} \quad (0 \le \theta \le \pi) \\ \arcsin \frac{y}{r} & \text{I, IV} \quad (-\pi/2 \le \theta \le \pi/2) \\ \arctan y/x + \pi & \text{quadrant III} \quad (\pi \le \theta \le 3\pi/2. \end{cases}$$

etc.

in reverse direction is easier to express:

$$\boxed{\begin{bmatrix} x \\ y \end{bmatrix} = r \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \begin{bmatrix} r\cos\theta \\ r\sin\theta \end{bmatrix}}$$

examples:

if $r=5$, $\theta=\pi/3$ $\quad \begin{bmatrix} x \\ y \end{bmatrix} =$

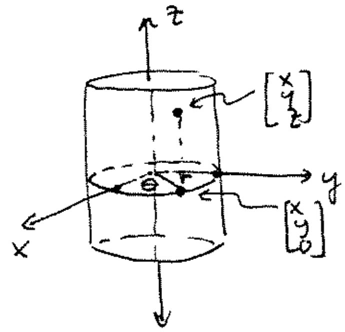
if $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -10 \\ 10 \end{bmatrix}$, $\begin{bmatrix} r \\ \theta \end{bmatrix} =$

draw the curve implicitly defined by $r=3$

draw the curve implicitly defined by $\theta = \frac{8}{3}\pi$ (& $r>0$).

Why are these coords called "polar coords"?

$\mathbb{R}^3$: cylindrical coordinates :



use $\begin{bmatrix} r \\ \theta \end{bmatrix}$ in the x-y plane, keep z

i.e. $\begin{bmatrix} r \\ \theta \\ z \end{bmatrix}$ $\}$ polar coords, $\}$ z

examples: What are the $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ ("rectangular") coords of a point with cylindrical coords $\begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}$?

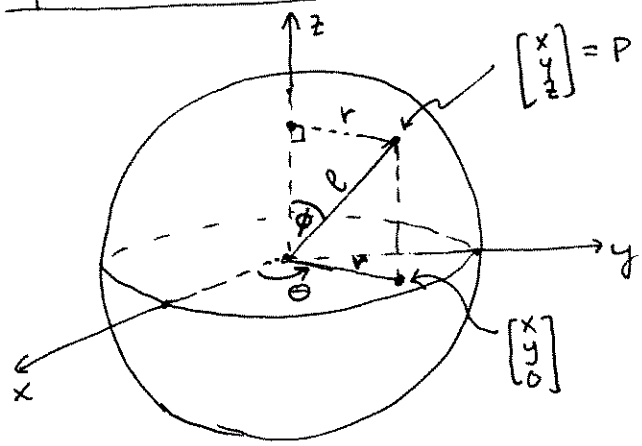
if $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ 4 \end{bmatrix}$ what are $\begin{bmatrix} r \\ \theta \\ z \end{bmatrix}$?

Sketch the surface implicitly defined by $r=3$. Why are these coords called cylindrical coords?

Sketch the surface implicitly defined by $\theta = \pi/4$ (& $r>0$)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} r\cos\theta \\ r\sin\theta \\ z \end{bmatrix}$$

spherical coordinates



$\rho := \sqrt{x^2+y^2+z^2}$

$\phi :=$ angle between pos z axis $(\hat{k})$ and $\vec{OP}$

$0 \le \phi \le \pi$ $\rightarrow$ so $\phi = \arccos\left(\frac{z}{\rho}\right)$

$\theta :=$ polar coord angle of projection $\begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$

so (see triangles in picture)

$z = \rho\cos\phi$

$r = \rho\sin\phi$ (polar r)

so $x = r\cos\theta$
$= \rho\sin\phi\cos\theta$

$y = r\sin\theta$
$= \rho\sin\phi\sin\theta$

examples

if $\begin{bmatrix} \rho \\ \theta \\ \phi \end{bmatrix} = \begin{bmatrix} 4 \\ \frac{5}{6}\pi \\ \frac{1}{3}\pi \end{bmatrix}$, What is $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$?

if $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$, what is $\begin{bmatrix} \rho \\ \theta \\ \phi \end{bmatrix}$?

sketch the surfaces implicitly defined by

(1) $\rho = 3$;

(2) $\theta = \pi/4$;

(3) $\phi = \pi/3$;

$$\boxed{\begin{aligned} x &= \rho\sin\phi\cos\theta \\ y &= \rho\sin\theta\sin\theta \\ z &= \rho\cos\phi \end{aligned}}$$

other examples:

What surface is implicitly defined
by the spherical coord eqtn
$$\rho = 2\cos\phi\,?$$
(hint: convert to rectangular coords)

What surface is implicitly defined by
the cylindrical coord eqtn
$$r^2 = z^2\,?$$