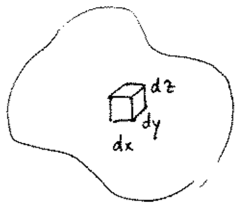
Math 2210-1
Friday Nov 12

§14.7 Triple integrals in spherical & cylindrical coords

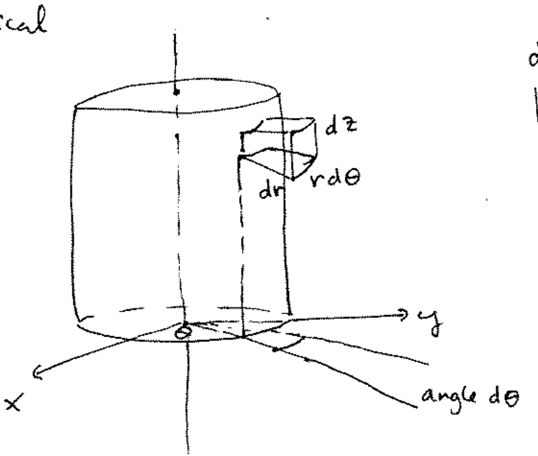
rectangular:



$dV = (dx)(dy)(dz)$

but you can partition a region using other coord systems, just like we used polar coords for double integrals

cylindrical coords



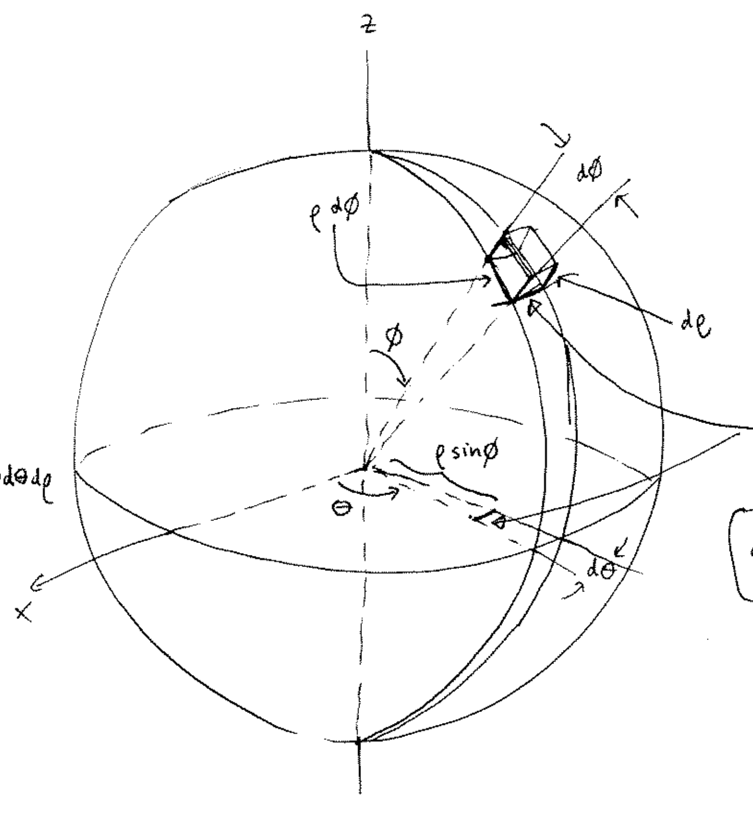
from polar coords
↓
$dV = (dA)\,dz$

$$\boxed{dV = r(dr)(d\theta)(dz)}$$

so $\displaystyle\iiint_R f(x,y,z)\,dV = \iiint f(r\cos\theta, r\sin\theta, z)\, r\,dr\,d\theta\,dz$

R specified in cylind. coords

spherical coords



$? = \rho\sin\phi\, d\theta$

$$\boxed{dV = \rho^2 \sin\phi\, d\phi\, d\theta\, d\rho}$$

so

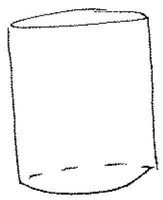
$$\iiint_R f(x,y,z)\,dV$$

$$= \iiint f\begin{pmatrix} \rho\cos\theta\sin\phi \\ \rho\sin\theta\sin\phi \\ \rho\cos\phi \end{pmatrix} \rho^2\sin\phi\, d\phi\, d\theta\, d\rho$$

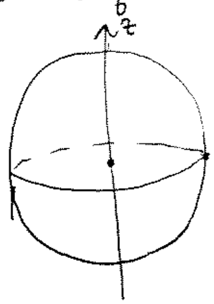
R specified in spherical coords

Examples

Volume of cylinder of radius a and height h:



Volume of ball of radius a



and its moment of inertia about z-axis, if density δ is const:

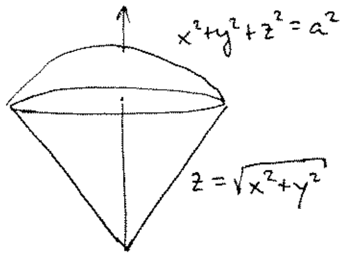
$$I_z = \iiint (x^2+y^2)\, \delta \, dV$$

(use 113 Int. table if nec.).

Example: Our friend the ice cream cone from old HW:

region inside $x^2+y^2+z^2=a^2$

and the ice-cream cone $z=\sqrt{x^2+y^2}$



Find volume & centroid ($\delta \equiv 1$).

ans $Vol = \frac{2}{3}\pi a^3 \left(1-\frac{1}{\sqrt{2}}\right)$.

$\bar{z} = \frac{3}{8(2-\sqrt{2})} a$

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> Vol:=int(int(int(rho^2*sin(phi),phi=0..Pi/4),rho=0..a),theta=0..2*
  Pi);
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$$Vol := \frac{2\left(-\frac{\sqrt{2}}{2}+1\right)a^3\pi}{3}$$

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> Mxy:=int(int(int(rho*cos(phi)*rho^2*sin(phi),phi=0..Pi/4),rho=0..a
  ),theta=0..2*Pi);
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$$Mxy := \frac{a^4\pi}{8}$$

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> zbar:=Mxy/Vol;
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$$zbar := \frac{3\,a}{16\left(-\frac{\sqrt{2}}{2}+1\right)}$$

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>
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