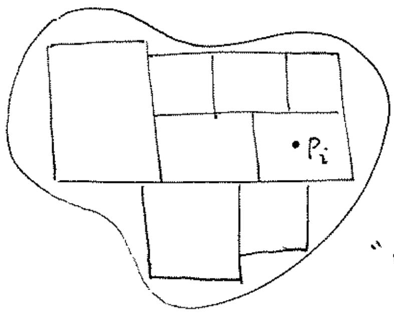
Math 2210-1
Monday 1 Nov. § 14.2

You can <u>define</u> double integrals
for "nice" domains in the plane,
and continuous functions defined on these domains



Riemann sum $\displaystyle\sum_{i=1}^{n} f(P_i)\,\underbrace{\Delta A_i}$

sample point in i^{th} rect.

area of i^{th} rectangle (<u>or</u> other geometric object).

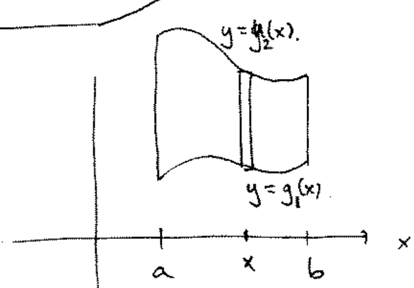
"inner partition" $\mathcal{P}$ for domain D.

the "norm" of the partition is defined to be the maximum "diameter" (diagonal distance) of ~~the~~ an rectangle in the partition.

In advanced Calculus, one studies the process by which Riemann sums converge to a limit, $\displaystyle\iint_D f(x,y)\,dA$ as the norm of the partitions $\to 0$.

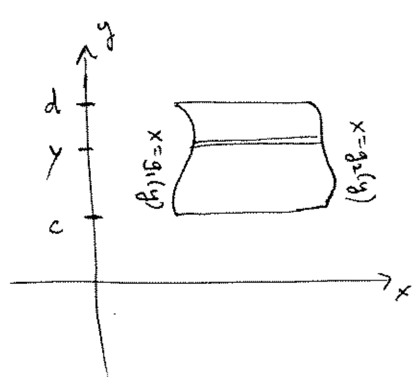
How to <u>compute</u> double integrals over non-rectangular domains

vertically simple region.



$$\iint_R f(x,y)\,dA = \int_a^b \left(\int_{g_1(x)}^{g_2(x)} f(x,y)\,dy \right) dx.$$

horizontally simple region



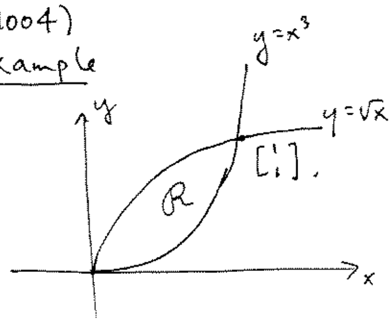
$$\iint_R f(x,y)\,dA = \int_c^d \left(\int_{g_1(y)}^{g_2(y)} f(x,y)\,dx \right) dy.$$

More complicated region: sub<u>divide</u>

this makes sense if you think about the volume interpretation of $\displaystyle\iint_R f\,dA$. (slicing).

(p. 1004)

Example



$f(x,y) = xy^2$

Compute $\iint\limits_R f(x,y)\,dA$ 2 ways (since region is vertically & horiz. simple)

Sketch as you go! (ans = $\frac{5}{77}$).

example

Find the region being integrated over in the following iterated integral. (Also, compute the integral.) Then compute the integral in the other order. (Which can be done!).

$$\int_{-2}^{1}\left(\int_{y^2}^{2-y} 2x \; dx\right) dy.$$

$\left(\text{ans} = \frac{72}{5}\right)$

Sometimes you need to reverse the order of integration in order to compute an integral.

example 4 p 1005

$$\int_0^2 \int_{y/2}^1 y e^{x^3} \, dx \, dy$$ (ans $\frac{2}{3}(e-1)$).