

Name.....
I.D. number.....

Math 2210-1
Exam 1
September 29, 2004

This exam is closed-book and closed-note. You may use a scientific calculator, but not one which is capable of graphing or of solving linear algebra equations. **In order to receive full or partial credit on any problem, you must show all of your work and justify your conclusions.** There are 100 points possible. The point values for each problem are indicated in the right-hand margin. **Good Luck!**

1a) Consider the following two lines, expressed parametrically:

$$\vec{r}(t) = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + t \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

$$\vec{q}(s) = \begin{bmatrix} 2 \\ -3 \\ -4 \end{bmatrix} + s \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

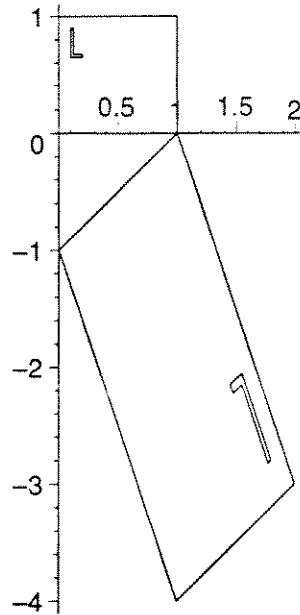
Find the point (which exists!) at which these two lines intersect.

(12 points)

1b) Find an (implicit) equation of the form $ax+by+cz=d$ for the plane containing the two lines in (1a). Notice that two parametric formulas contain enough information for you to do this problem without having completed (1a).

(12 points)

2) Consider an affine map from R^2 to R^2 . Consider the following L-box diagram, which shows the unit "L-box" and where it is transformed to by our mystery affine map.



2a) Find the formula for the mystery affine map.

(15 points)

2b) What is the area of the image L-box?

(5 points)

3) Consider the parametric curve given by

$$\vec{r}(t) = \begin{bmatrix} e^t \\ e^{-2t} \end{bmatrix}$$

There is a sketch of part of the range of \vec{r} on the next page.

3a) Do a short computation to show that the range of \vec{r} lies on the graph $y = \frac{1}{x^2}$, in the x-y plane.

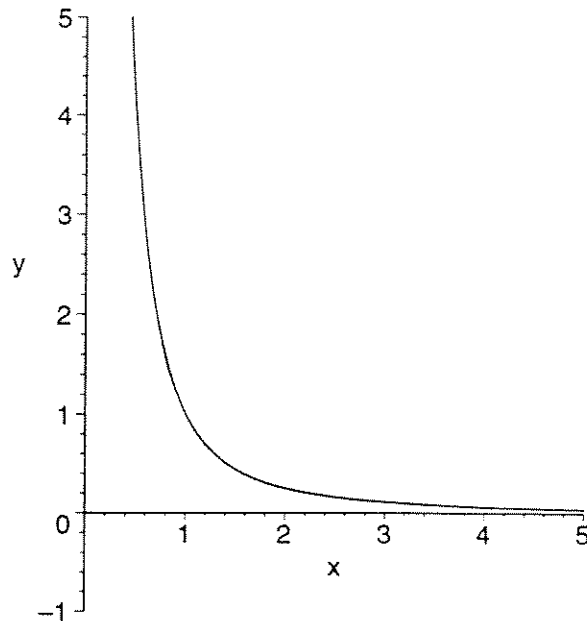
(4 points)

3b) Since the curvature of a curve is a property of the curve and not how it is parameterized, you can use the formula for the curvature of a graph $y = f(x)$ to compute the curvature at the image point (1,1). Do this, using the formula displayed on the blackboard.

(6 points)

3c) Returning to the parameterized version of the curve, compute $\vec{r}'(t)$ and $\vec{r}''(t)$.

(4 points)



3d) Label the point with position vector $\vec{r}(0)$ into the picture above. Compute, and accurately draw the vectors $\vec{r}'(0)$ and $\vec{r}''(0)$ into the picture, in the appropriate location(s). Also, compute and draw in the unit tangent and normal vectors, \vec{T} and \vec{N} , when $t=0$. (Hint: you can find \vec{N} easily because it is perpendicular to \vec{T} and the curve lies in a plane.)

(10 points)

3e) Use your sketch, and an appropriate right triangle which you add to it, to crudely estimate (accuracy within 0.5 suffices) scalars a and b so that

$$\vec{r}''(0) = a\vec{T} + b\vec{N}$$

(4 points)

Do ONE of (3f), (3g). If you try both ways, indicate which one you would like graded:

3f) You will now find the exact values of a and b : First, explain why the (3e) equation implies that

$$a = \vec{r}''(0) \cdot \vec{T}$$

$$b = \vec{r}''(0) \cdot \vec{N}$$

Then compute the dot products to recover a and b . (Hopefully your exact values for a and b are consistent with your numerical estimates from (3e).

(12 points)

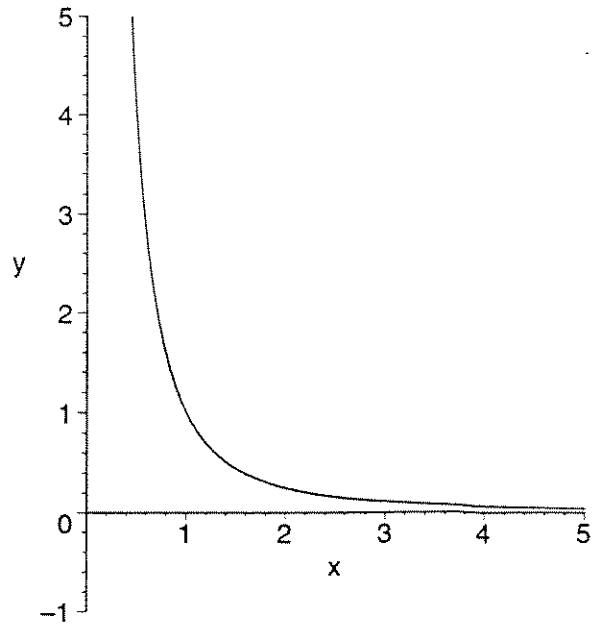
OR

3g) Another way to get a and b is to use the acceleration decomposition formula

$$\vec{r}''(t) = \left(\frac{d}{dt} v(t) \right) \vec{T} + \kappa v^2 \vec{N}$$

where the scalar v represents speed and κ is the curvature. Using your curvature computation in (3b), and computing the speed and it's derivative at $t=0$, compute the values for a and b in the equation from (3e).

(12 points)



(spare picture and space)