Math 2210
Fri 27 Aug

Problem session room:
Tuesdays 9:40-10:30
LCB 121

- vector addition and scalar multiplication (page 2 Wed.)

For $\vec{u} = \langle u_1, u_2 \rangle$
$\vec{v} = \langle v_1, v_2 \rangle$
$t \in \mathbb{R}$

Then

$\vec{u} + \vec{v} :=$

$t\vec{u} =$

geometric pictures:

arithmetic properties

$\vec{u} + \vec{v} = \vec{v} + \vec{u}$ + is commutative

$\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$ + is associative

$t(\vec{u} + \vec{v}) = t\vec{u} + t\vec{v}$

$(s+t)\vec{u} = s\vec{u} + t\vec{u}$ } distributive properties

$(st)\vec{u} = s(t\vec{u}) = t(s\vec{u})$ · associative

Check one of these properties (more on HW):

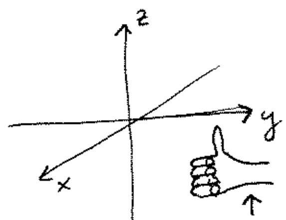
- page 3 Wed

$\mathbb{R}^3 = \{ (x,y,z) \text{ s.t. } x \in \mathbb{R} \}$

$\mathbb{R}^3$ (3-dim'l Euclidean space)

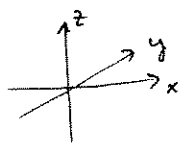
"is the set of all ordered triples $(x,y,z)$ such that $x$ is an element of the real numbers"

geometrically, we represent $\mathbb{R}^3$ using right-handed coord systems
(actually we used "right-handed" coords for $\mathbb{R}^2$ as well.)




your right hand.

or

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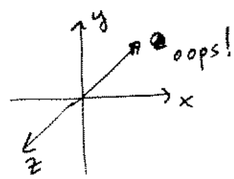
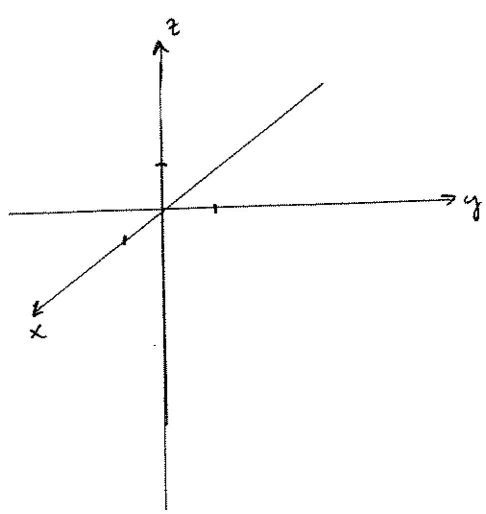



to see where points are it often helps to plot boxes for which the points are corners

example

plot $(1,2,-3)$




example

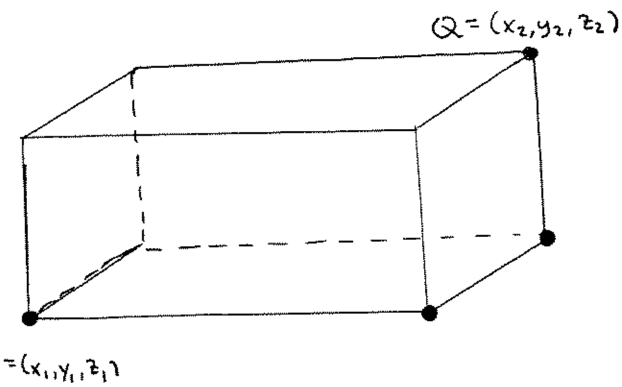
if $P = (x_1, y_1, z_1)$
$Q = (x_2, y_2, z_2)$

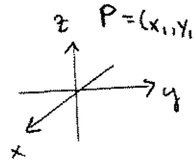
then distance from $P$ to $Q$ is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

two applications of Pythagorean Thm:

Details:



<u>example</u>: (example 3 p. 782)

What geometric object is described by the set of points $(x,y,z)$ satisfying the equation

$$x^2 + y^2 + z^2 + 4x + 2y - 6z - 2 = 0 \, ?$$

<u>Vectors in 3-space</u>

<u>Def</u>: a 3-vector is an ordered triple $\vec{v} = \langle v_1, v_2, v_3 \rangle$ of real numbers

<u>Def</u>: length (or magnitude) of $\vec{v}$, $|\vec{v}| := \sqrt{v_1^2 + v_2^2 + v_3^2}$

is the length of the directed line segment determined by (i.e. with displacement) $\vec{v}$.

<u>Def</u>: $\overrightarrow{PQ}$, $\overrightarrow{OP}$

<u>Def</u>: If $\vec{u} = \langle u_1, u_2, u_3 \rangle$
$\vec{v} = \langle v_1, v_2, v_3 \rangle$
$t \in \mathbb{R}$

then $\vec{u} + \vec{v} =$
$t\vec{u} =$

same properties and geometric interpretation as page 1 !

$\vec{i} = \langle 1,0,0 \rangle$
$\vec{j} = \langle 0,1,0 \rangle$
$\vec{k} = \langle 0,0,1 \rangle$

<u>example</u> $\vec{a} = \langle 3,4,12 \rangle$
$\vec{b} = \langle -4,3,0 \rangle$

$\vec{a} + \vec{b} =$

$|\vec{a}| =$

$3\vec{a} =$

$3\vec{a} - 2\vec{b} =$

draw a directed line segment representing $\vec{a}$, and illustrate

$\vec{a} = 3\vec{i} + 4\vec{j} + 12\vec{k}$.

On Monday we will discuss the <u>dot product</u> of vectors

$$\langle u_1, u_2, u_3 \rangle \cdot \langle v_1, v_2, v_3 \rangle = u_1 v + u_2 v_2 + u_3 v_3 \qquad \text{(a scalar!)}.$$

algebra properties:

$$\vec{u}\cdot\vec{u} = |\vec{u}|^2$$

$$\vec{u}\cdot\vec{v} = \vec{v}\cdot\vec{u}$$

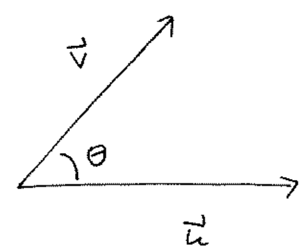
$$\vec{u}\cdot(\vec{v}+\vec{w}) = \vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{w}$$

$$\left(\text{so } (\vec{u}+\vec{v})\cdot\vec{w} = \vec{u}\cdot\vec{w} + \vec{v}\cdot\vec{w}\right)$$

$$(t\vec{u})\cdot\vec{v} = t(\vec{u}\cdot\vec{v}) = \vec{u}\cdot(t\vec{v})$$

geometric property:

$$\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta$$




Lets you find $\cos\theta$ algebraically. <u>Extremely</u> useful

Math 2210-1
Homework due 9/1 : Hand in circled problems

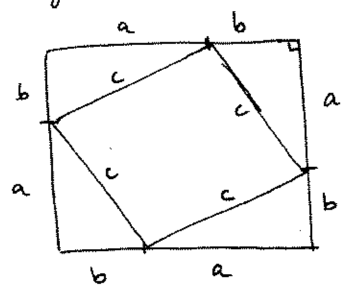
11.1 1 2 5 6 14 19 24 29 45 51 53

11.2 5, 11, 17, 25, 45, 50, 53, 56, 57, 69, 70

And

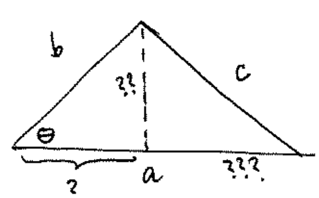
I The Pythagorean Theorem says that for a right $\Delta$, with legs a, b and hypotenuse c,
$$c^2 = a^2 + b^2.$$

Use the following diagram (by computing the area of the $(a+b) \times (a+b)$ square two ways) to prove Pythagoreas Theorem: Hint: first show the inside $\square$ is a square.




II Carefully check one of the 5 arithmetic properties on page 1 of notes, using 3-vectors. Don't check the one we did in class

III Prove the law of cosines for an acute angle of a triangle, using 2 Pythagorean Thm applications, as indicated by diagram:




$$\boxed{c^2 = a^2 + b^2 - 2ab\cos\theta}$$

(we use this fact to understand geometric meaning of dot product).

IV For $\vec{a} = \langle a_1, a_2, a_3 \rangle$
$\vec{b} = \langle b_1, b_2, b_3 \rangle$
$\vec{c} = \langle c_1, c_2, c_3 \rangle$

Show that the dot product distributes over addition:
$$\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}.$$