Math 2210
Wed 25 Aug

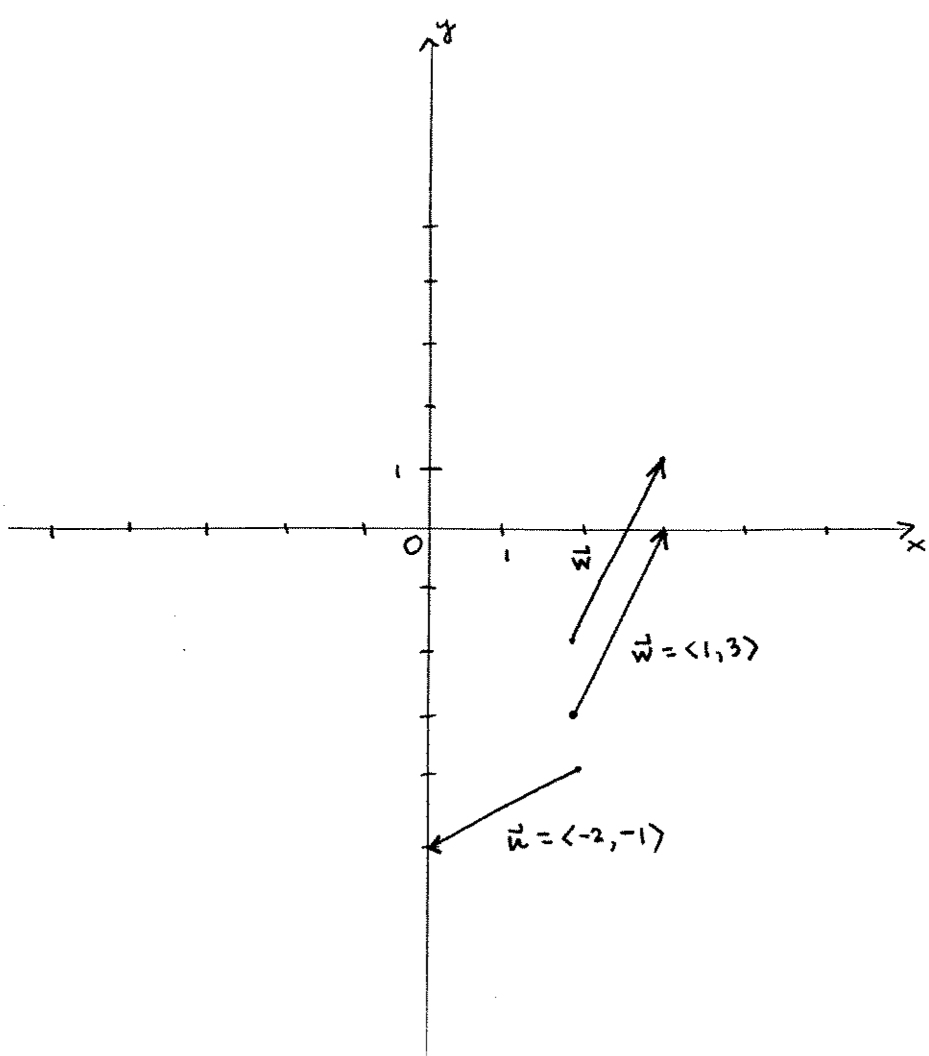
HW 11.1 1, 2, 5, 6, 14, 19, 24, 29, 45, 51, 53

§11.1 (11.2) Vectors.

Def A vector $\vec{v}$ in the Cartesian (Euclidean) plane $\mathbb{R}^2$, is an ordered pair of numbers $\vec{v} = \langle a, b \rangle$.

a and b are called the (1^{st} & 2^{nd}) components of $\vec{v}$

geometric meaning: We use vectors to represent displacement associated to directed line segments, and if $\vec{v} = \langle a, b \rangle$, a would correspond to x-displacement, b to y-displacement.



If P, Q are points $\boxed{\vec{v} = \overrightarrow{PQ}}$ is the displacement vector from P to Q.
$\overrightarrow{OP}$ is the position vector of the point P
(O is the origin).

Class exercise

Let $P = (4,3)$
$Q = (-2,5)$

(1) Plot P, Q
(2) Draw $\overrightarrow{OP}, \overrightarrow{OQ}, \overrightarrow{PQ}$
(3) compute $\overrightarrow{PQ}$

(4) If $R = (x_1, y_1)$
$S = (x_2, y_2)$
What is $\overrightarrow{RS}$?

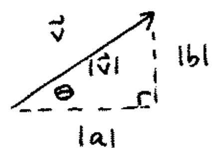
(5) If you go from (100 E, 200 N) to (300 W, 500 S) in SLC, what is your displacement vector?

vectors have length and direction.

if $\vec{v} = \langle a, b \rangle$

$|\vec{v}| := \sqrt{a^2+b^2}$

length ("magnitude") of $\vec{v}$



θ = polar angle, $\cos\theta = \frac{a}{|\vec{v}|}$

i.e. angle between $\vec{v}$ and positive x-direction. $\sin\theta = \frac{b}{|\vec{v}|}$

(5) on page 1, $|\vec{PQ}| =$

<u>Algebra of vectors</u>

Let $\vec{u} = \langle u_1, u_2 \rangle$

$\vec{v} = \langle v_1, v_2 \rangle$

<u>Defs</u>: $\vec{u} = \vec{v}$ if and only if (iff) $u_1 = v_1$ and $u_2 = v_2$

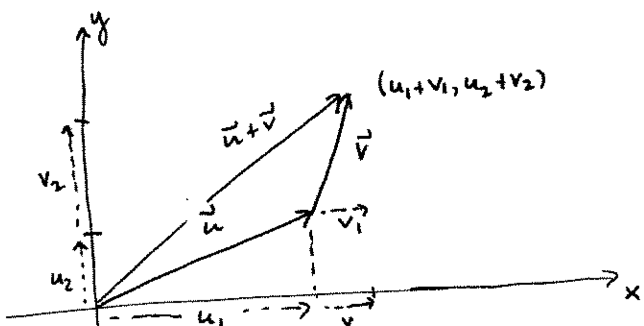
$\vec{u}+\vec{v} := \langle u_1+v_1, u_2+v_2 \rangle$ <u>vector addition</u>

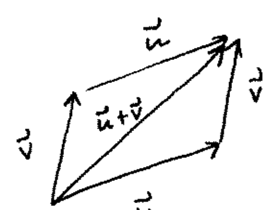
$t \in \mathbb{R} \Rightarrow t\vec{u} := \langle tu_1, tu_2 \rangle$ <u>scalar multiplication</u>

easy to check arithmetic:

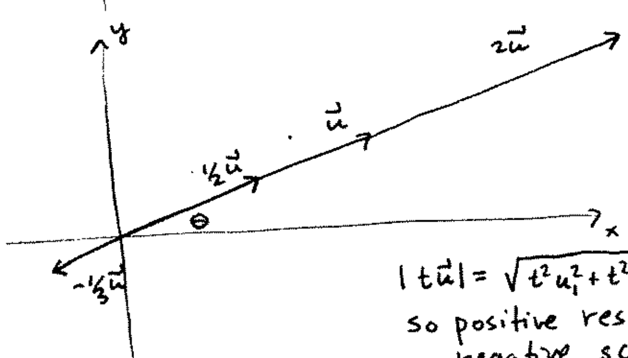
$$\left\{\begin{array}{l} \vec{u}+\vec{v} = \vec{v}+\vec{u} \\ \vec{u}+(\vec{v}+\vec{w}) = (\vec{u}+\vec{v})+\vec{w} \\ t(\vec{u}+\vec{v}) = t\vec{u}+t\vec{v} \\ (s+t)\vec{u} = s\vec{u}+t\vec{u} \\ (st)\vec{u} = s(t\vec{u}) = t(s\vec{u}) \end{array}\right.$$

geometric meaning : vector addition computes net displacement
scalar multiplication rescales.



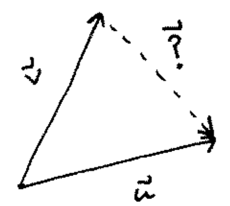


geometric interpretation of $\vec{u}+\vec{v} = \vec{v}+\vec{u}$



$|t\vec{u}| = \sqrt{t^2u_1^2+t^2u_2^2} = |t||\vec{u}|$

so positive rescalings point in same direction (θ)
negative scalar multiples point in <u>opposite</u> direction ($\theta+\pi$)

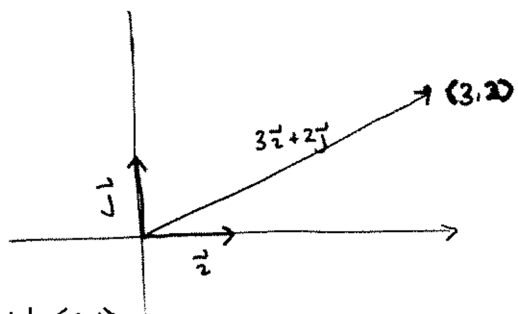


$$\vec{v} + \vec{?} = \vec{u}$$
$$\vec{?} = \vec{u} - \vec{v}. \quad !$$

Special vectors

$\vec{i} := \langle 1, 0 \rangle$

$\vec{j} := \langle 0, 1 \rangle$

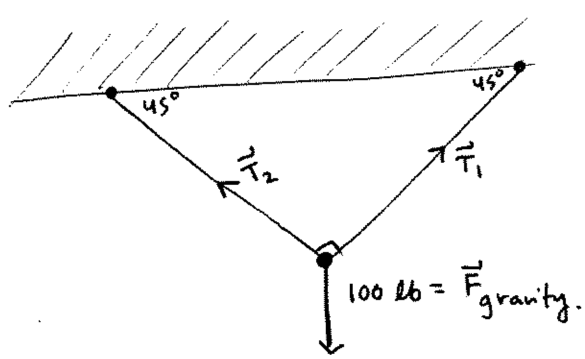


then

$$\vec{v} = \langle a, b \rangle = a\langle 1, 0 \rangle + b\langle 0, 1 \rangle$$
$$= a\vec{i} + b\vec{j}$$

<u>Force vectors</u> : Newton's law says (implies) that for mechanical systems in equilibrium, net forces at any pt of the system are zero.

<u>Example 6 p.778</u>



Find $\vec{T}_1$ and $\vec{T}_2$

$|\vec{T}_1|, |\vec{T}_2|$

trig $\Rightarrow$ $\vec{T}_1 = \langle a, a \rangle \quad (a > 0)$

$\vec{T}_2 = \langle -a, a \rangle$

$\vec{F} = \langle 0, -100 \rangle$

Newton $\Rightarrow$ $\vec{T}_1 + \vec{T}_2 + \vec{F} = \vec{0}$

$$\underbrace{\vec{T}_1 + \vec{T}_2}_{\langle 0, 2a \rangle} = -\vec{F} = \langle 0, 100 \rangle$$

$\Rightarrow a = 50$

so what is $|\vec{T}_1|, |\vec{T}_2|$?