MATHEMATICS 2210-1
Calculus III - Multivariable Calculus
Fall semester 2004

text: *Multivariable Calculus with Matrices, 6e*
by C.H. Edwards Jr. and David E. Penney
when: MWF 9:40-10:30
where: LCB 219

instructor: Prof. Nick Korevaar
office: LCB 204
telephone: 581-7318
email: korevaar@math.utah.edu
office hours: M, W, F 10:35-10:55 a.m., T 11-11:50 a.m., 1:00-1:50 p.m. and by appointment.

2210-1 home page: www.math.utah.edu/ ~korevaar/2210fall04

prerequisites: Math 1210 and 1220 or equivalent (for example, an AP score of at least 3 on the BC Calculus exam). For our Department's placement recommendations, visit www.math.utah.edu/ugrad/ap.html.

course outline: This is the final course in the three-semester Calculus sequence, Mathematics 1210-1220-2210. As you have already been learning, Calculus is part of the mathematical foundation with which science can model the world. Isaac Newton (1642-1727) was one of its co-discoverers, and his aim was to understand the physics he saw in the natural world, such as planetary motion. A beautiful quote of Galileo, from 1623, anticipates the mathematics which has followed:

> *Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it.*

In single-variable Calculus the domain and range (input and output) of the functions you consider are both subsets of the real numbers. In real-world problems it is often more natural to consider multivariable possibilities, for the domain, the range, or both. Thus, if you want to model planetary (or other) motion, the input can be the real variable t, for time, but the output position function should either describe points in space (i.e. 3-dimensional real space, $\mathbb{R}^3$), or in a plane $\mathbb{R}^2$ containing the sun and the planet. If you want to study the temperature in Utah, the input could be described by (x, y, t), where (x, y) are used to describe where you are in Utah, and t is time, and the output temperature $T(x, y, t)$ is a real number. If you want to describe electrical or magnetic fields, or complete weather systems, or any complicated system in science, engineering, business, medicine or industry, then the inputs and ouputs are usually both multivariable. Our goal in Math 2210 is to adapt and extend the ideas of the derivative, the integral, and the Fundamental Theorem of Calculus to multivariable settings, and to study important applications which result.

It is a good idea to understand the geometry and algebra of the plane $\mathbb{R}^2$, 3-space $\mathbb{R}^3$, and n-space $\mathbb{R}^n$, in order to understand functions between them, and so we do this at the start of the course, beginning with Chapter 11 of the text, *Vectors and Matrices*. In fact, vector and matrix algebra are integrated into the remainder of the course material, chapters 12-15, and this is the reason why our section of 2210, as well as section 2, are experimenting with this Edwards-Penney text.

If you don't want to change texts from 1210-1220, you might consider sections 3 or 4 of Math 2210; those sections are using the more standard treatment from chapters 13-17 of the 1210-1220 text by Varberg.

Chapter 12 is primarily about functions with real-number input and multivariable ($\mathbb{R}^n$) output, such as those which describe particle motion. This is an important context to consider, and one you will return to in Math 2250 or 2280, when you consider systems of ordinary differential equations. The primary calculus objects we consider will be the tangent (velocity) and acceleration vectors associated to particle motion.

Chapter 13 is a study of differential calculus for functions when the domain is multivariable. The notion of derivative for such functions has a new twist based on linear approximation. We will meet new versions of the chain rule, critical points for optimization problems, and the second derivative test, and these versions will require matrix and vector geometry. Many of you will study functions with multivariable domains in courses about partial differential equations, for example Math 3150.

Chapter 14 is a study of definite integration for real valued functions of several variables, i.e. when the domain is a subset of the plane or 3-space rather than just an interval in the real numbers. It will turn out that we can reduce these integral problems to iterated 1-variable integrals, at which you are already experts. To understand how to change of variables, however, we will appeal again to our chapter 11 understanding of matrix algebra and geometry.

Chapter 15, Vector Calculus, is analagous to the Fundamental Theorem of Calculus and its applications in 1210-1220. There are various versions of these generalizations, and they all relate integrals of certain (partial) derivatives of functions over domains in $\mathbb{R}^n$, to boundary integrals. Included in this zoo of results are Green's, Gauss', and Stoke's Theorems, which are the foundation of classical physics topics such as electro-magnetism and fluid mechanics.

grading: There will be three midterms, a comprehensive final examination, and homework. Each midterm will count for 15% of your grade, homework will count for 30%, and the final exam will make up the remaining 25%.

Homework will be assigned daily and collected weekly, on Wednesdays. Our grader will grade a subset of the problems you hand in. You are strongly encouraged to work together on homework problems, and to use whatever technology you find helpful. You must each complete and hand in your own problem set, however. I use homework problems to let you drill basic skills, but also to explore more in-depth problems and applications. The value of carefully working homework problems is that mathematics (like anything) must be practiced and experienced to be learned, so make sure that you really understand each problem you hand in.

I will arrange for an optional Tuesday problem session, around this class time, to which you are all invited. I will let you know the precise meeting time and location soon. In addition to this special problem session, the Math Department Tutoring Center is located in Rushing Student Center and is open for free tutoring from 8 a.m. to 8 p.m. on M-Th, and from 8 a.m. to 6 p.m. on Friday. Some, but not all of the math tutors welcome questions from Math 2210 students. To see the times and specialities of various tutors, consult the web address www.math.utah.edu/ugrad/tutoring.html .

It is the Math Department policy, and mine as well, to grant any withdrawl request until the University deadline of Friday October 22.

ADA statement: The American with Disabilities Act requires that reasonable accomodations be provided for students with physical, sensory, cognitive, systemic, learning, and psychiatric disabilities. Please contact me at the beginning of the semester to discuss any such accommodations for the course.

Tentative Daily Schedule

exam dates fixed,
daily subject matter approximated

W	25 Aug	11.1-11.2	vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$
F	27 Aug	11.2	dot product algebra and geometry
M	30 Aug	11.3	cross products and determinants
W	1 Sept	11.4	lines and planes in space
F	3 Sept	11.5	linear systems and matrices
M	6 Sept	none	Labor Day
W	8 Sept	11.6	matrix algebra and geometry
F	10 Sept	11.6	continued
M	13 Sept	11.7	eigenvalues and eigenvectors
W	15 Sept	12.1	curves and motion in space
F	17 Sept	12.2	curvature and acceleration
M	20 Sept	12.2	continued
W	22 Sept	12.3	cylinders and quadric surfaces
F	24 Sept	12.4	polar, cylindrical, spherical coordinates
M	27 Sept	13.2	functions of several variables
W	29 Sept	Exam 1	chapters 11-12
F	1 Oct	13.3	limits and continuity
M	4 Oct	13.4	partial derivatives
W	6 Oct	13.5	optimization problems
F	8 Oct	none	fall break day
M	11 Oct	13.6	affine approximations and derivative matrices
W	13 Oct	13.7	multivariable chain rule
F	15 Oct	13.7	inverse and implicit functions
M	18 Oct	13.8	directional derivatives and the gradient vector
W	20 Oct	13.9	Lagrange multipliers
F	22 Oct	13.10	second derivative test
M	25 Oct	14.1	double integrals
W	27 Oct	Exam 2	chapter 13
F	29 Oct	14.2	double integrals over general regions

M	1 Nov	14.3	area and volume by double integration
W	3 Nov	14.4	double integrals in polar coordinates
F	5 Nov	14.5	applications of double integrals
M	8 Nov	14.6	triple integrals
W	10 Nov	14.7	integrals in cylindrical and spherical coordinates
F	12 Nov	14.8	surface area
M	15 Nov	14.9	change of variables in multiple integrals
W	17 Nov	15.1	vector fields
F	19 Nov	Exam 3	chapter 14
M	22 Nov	15.2	line integrals
W	24 Nov	15.3	path independence and conservative vector fields
F	26 Nov	none	Thanksgiving recess
M	29 Nov	15.4	FTC and Green's Theorem
W	1 Dec	15.5	surface integrals
F	3 Dec	15.6-15.7	FTC, divergence, Stoke's Theorems
M	6 Dec	15.6-15.7	continued
W	8 Dec	11-15	review
F	10 Dec	none	University reading day
Thurs	16 Dec	FINAL EXAM	entire course 8-10 a.m.