Math 1210-1
Quiz 9 Solutions
April 15 2016

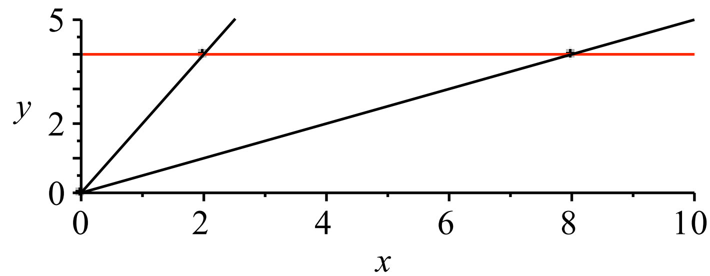
1a) Sketch the triangle bounded by the lines $y = \frac{1}{2}x$, $y = 2\,x$, $y = 4$. Find the coordinates of the three vertices. Compute the triangle area using $A = \frac{1}{2}bh$. (Use the "base" on the top of the triangle.)

(4 points)

solution: The points of intersection between the two diagonal lines and the horizontal line y = 4 are when

$$\frac{1}{2}x = 4 \Rightarrow x = 8 \Rightarrow (8, 4)$$
$$2\,x = 4 \Rightarrow x = 2 \Rightarrow (2, 4).$$

the lines $y = \frac{1}{2}x$, $y = 2\,x$ *intersect when* $x = 0 \Rightarrow (0, 0)$.



The length of the "base" on the top is 8-2=6, the height is 4, so the area is

$$A = \frac{1}{2}bh = \frac{1}{2}\cdot 6\cdot 4 = 12.$$

1b) Use Calculus to recompute the triangle area. You may either use vertical chopping - which will lead to the sum of two "dx" integrals, or horizontal chopping - which will lead to a single "dy" integral. If you correctly do both methods you will receive 4 extra credit points. (Use the back of the page if necessary.)

(6 points)

solution: *Using y as the variable, the triangle is described by* $0 \le y \le 4$ *with y varying from* $x = \frac{y}{2}$ *to* $x = 2\,y$. *So, a typical rectangle area in a Riemann sum would be the area of a rectangle* Δy *thick, with sideway's height from the point* $\left(\frac{y}{2}, y\right)$ *to* $(2\,y, y)$ *equal to* $2\,y - \frac{y}{2}$:

$$\Delta A = \left(2\,y - \frac{y}{2}\right)\Delta y = \frac{3}{2}y(\Delta y)$$

$$A = \int_0^4 \frac{3}{2}y\,\mathrm{dy} = \frac{3}{2}\left[\frac{y^2}{2}\right]_0^4 = \frac{3}{2}[8 - 0) = 12$$

To do this as a "dx" integration we must break the $x-$ *interval into two pieces,* $0 \leq x \leq 2$, $2 \leq x \leq 8$.

For $0 \leq x \leq 2$, *vertical Riemann sum rectangles with vertices at* $\left(x, \frac{1}{2}x\right)$, $(x, 2x)$ *have*

$$\Delta A = \left(2x - \frac{1}{2}x\right)\Delta x = \left(\frac{3}{2}x\right)\Delta x$$

For $2 \leq x \leq 8$, *vertical Riemann sum rectangles with vertices at* $\left(x, \frac{1}{2}x\right)$, $(x, 4)$ *have*

$$\Delta A = \left(4 - \frac{1}{2}x\right)\Delta x.$$

So, the total area is

$$A = \int_0^2 \frac{3}{2}x \, dx + \int_2^8 4 - \frac{1}{2}x \, dx$$

$$= \frac{3}{4}\left[x^2\right]_0^2 + \left[4x - \frac{1}{4}x^2\right]_2^8$$

$$= \frac{3}{4}[4-0] + [(32-16)-(8-1)]$$

$$= 3 + 9 = 12.$$