Math 1210-1
Quiz 5 SOLUTIONS
February 19, 2016

Directions: You may ask and answer each others questions on this quiz. The goal is to understand what you're doing and express your thoughts clearly. Write your own solutions though, rather than just copying someone elses. Calculators are not allowed on this quiz. Show your work.

1) Consider the slanted ellipse with equation

$$2x^2 + 2xy + y^2 = 8.$$

1a) (4 points) Find y' in terms of x, y, using implicit differentiation.
solution: We are assuming $y = y(x)$. So we differentiate the identity above, and then solve for y':

$$\begin{aligned} 4x + 2(1\cdot y + x\cdot y') + 2yy' &= 0 \\ y'(2x + 2y) &= -4x - 2y \\ y' &= -\frac{4x - 2y}{2x + 2y} = \frac{2x - 2y}{x + y}. \end{aligned}$$

1b) (4 points) Find the equation of the line tangent to the ellipse at the point $(-2, 0)$. Write the line equation in slope-intercept form.
solution: The slope of the tangent line is given by y' at the point $(-2, 0)$. Plugging $x = 2, y = 0$ into the formula from part a,

$$y' = -\frac{4 - 0}{-2 + 0} = -2.$$

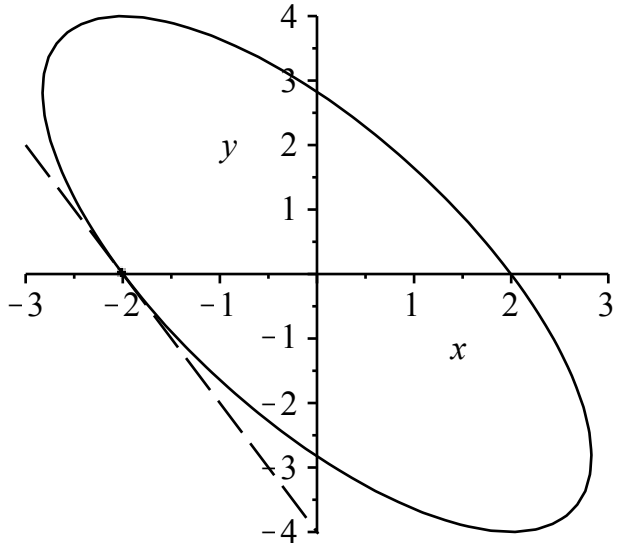
Thus, the point-slope form of the tangent line is

$$\begin{aligned} y - y_0 &= m(x - x_0) \\ y - 0 &= -2(x + 2) \end{aligned}$$

which has slope-intercept form

$$y = -2x - 4.$$

1c) (2 points) Sketch your tangent line from 1b onto the graph of the ellipse below, as a way to geometrically check your algebraic computations in parts a,b.


4
3
2
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y
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