

Math 1210-1
Quiz 4 SOLUTIONS
February 12, 2016

Directions: You may ask and answer each others questions on this quiz. The goal is to understand what you're doing and express your thoughts clearly. Write your own solutions though, rather than just copying someone elses. Calculators are not allowed on this quiz. Show your work.

1) (3 points) Use the quotient rule to find $D_x \frac{\sin(x)}{\cos(x)}$. Simplify your answer to show that your computation reproduces the derivative shortcut for $D_x \tan(x)$.

Solution:

$$D_x \frac{\sin(x)}{\cos(x)} = \frac{f'g - fg'}{g^2} = \frac{\cos(x)\cos(x) - \sin(x)(-\sin(x))}{\cos^2(x)} = \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)} = \sec^2(x)$$

2a) (2 points) Find $f'(x)$ for $f(x) = \sqrt{x^2 + 15}$.

Solution:

$$f(x) = (x^2 + 15)^{\frac{1}{2}}$$

so $f(x) = h(g(x))$ with $g(x) = x^2 + 15$, $h(u) = u^{\frac{1}{2}}$. Since $g'(x) = 2x$ and $h'(u) = \frac{1}{2}u^{-\frac{1}{2}}$ the chain rule yields

$$f'(x) = h'(g(x))g'(x) = \frac{1}{2}(x^2 + 15)^{-\frac{1}{2}} 2x = \frac{x}{\sqrt{x^2 + 15}}.$$

2b) (2 points) Find the equation of the line tangent to the graph of $f(x) = \sqrt{x^2 + 15}$, at the point on the graph with $x = 1$.

Solution: From part 2a, $f'(1) = \frac{1}{\sqrt{16}} = \frac{1}{4}$; this is the slope of the tangent line. The line passes

through the point $(1, f(1)) = (1, 4)$ so has equation

$$y - 4 = m(x - 1)$$

$$y - 4 = \frac{1}{4}(x - 1)$$

which can alternately be simplified into slope-intercept form

$$y = \frac{1}{4}x + 4 - \frac{1}{4} = \frac{1}{4}x + \frac{15}{4}.$$

3) (3 points) Find

$$D_t \frac{(3t^2 + 1)^{11}}{\cos(2t)}.$$

Solution:

$$D_t \frac{(3t^2 + 1)^{11}}{\cos(2t)} = \frac{f'g - fg'}{g^2} = \frac{11(3t^2 + 1)^{10} \cdot 6t \cdot \cos(2t) - (3t^2 + 1)^{11} (-\sin(2t) \cdot 2)}{\cos^2(2t)}.$$