Math 1210-1
Quiz 2 SOLUTIONS
January 22, 2016

Directions: You may ask and answer each others questions on this quiz. The goal is to understand what you're doing and express your thoughts clearly. Write your own solutions though, rather than just copying someone elses. Calculators are not allowed on this quiz. Show your work.

1) Let $f(x)$ be defined by

$$f(x) = \frac{x+2}{x^2+5x+6}.$$

a) At which points is f discontinuous? Classify each discontinuity as removable or not removable.

(6 points)

solution: rational functions are only discontinuous at points where the denominator equals zero. Since the denominator factors:

$$x^2+5x+6 = (x+2)(x+3)$$

$f(x)$ is undefined and therefore discontinuous, at $x=-2,-3$ (and continuous everywhere else).
The discontinuity at $x=-2$ is removable because

$$\lim_{x\to -2} f(x) = \lim_{x\to -2} \frac{x+2}{(x+2)(x+3)}$$

$$\lim_{x\to -2} \frac{1}{x+3} = 1$$

so we could make f continuous at $x=-2$ be defining $f(-2)=1$.
The discontinuity at $x=-3$ is not removable, because

$$\lim_{x\to -3} f(x) = \lim_{x\to -3} \frac{1}{x+3}$$

does not exist.

b) Find $\lim_{x\to\infty} f(x)$

(2 points)

solution:

$$\lim_{x\to\infty} \frac{x+2}{x^2+5x+6}\,\frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty} \frac{\frac{1}{x}+\frac{2}{x^2}}{1+\frac{5}{x}+\frac{6}{x^2}} = \frac{0+0}{1+0+0} = 0.$$

c) Find $\lim_{x\to -3^+} f(x)$.

(2 points)

solution:

$$\lim_{x\to -3^+} \frac{1}{x+3} = \frac{1}{0^+} = \infty.$$

(In other words, for $x > -3$, $x \approx -3$, $x+3 > 0$, but $x+3 \approx 0$, so $\frac{1}{x+3}$ is a large positive number,

which can be made arbitrarily large by choosing x close enough to −3.)