Name Solutions

Student I.D. _____

Math 1210-1
Quiz 10
April 22, 2016

Directions: You may ask and answer each others questions on this quiz. The goal is to understand what you're doing and express your thoughts clearly. Write your own solutions though, rather than just copying someone elses. Calculators are not allowed on this quiz. Show your work.

1a) Sketch the region in the first quadrant bounded below by the parabola $y = \frac{1}{2}x^2$ and above by the line $y = 2x$. Label the points where these two curves intersect (2 points)

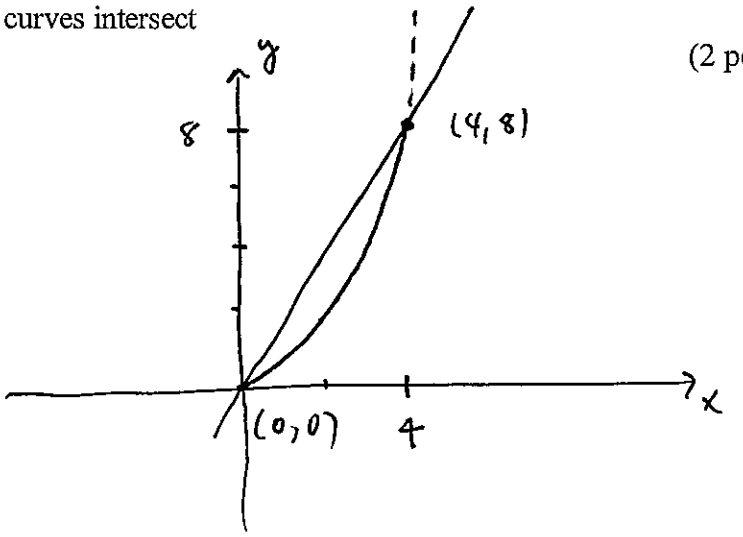
intersection: $\frac{1}{2}x^2 = 2x$

$x^2 = 4x$

$x^2 - 4x = 0$

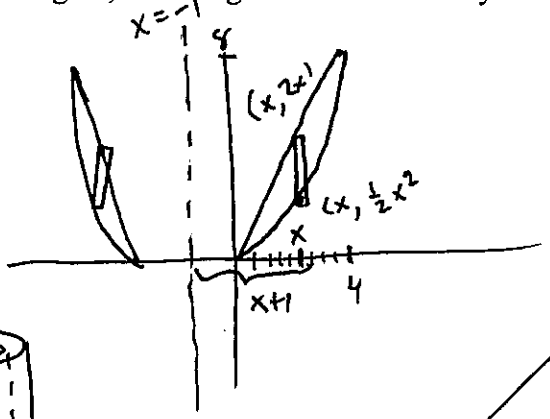
$x(x-4) = 0$

$x=0$, $y=0$ $\quad$ $x=4$, $y=8$



1b) Consider the solid obtained by rotating the region the region in 1a about the line $x = -1$. Compute the volume of this region, using cylindrical shells. For up to 4 extra credit points recompute the volume integral of the same region, but using washers. You may use the back of the page if necessary. (8 points)

shells:



$\Delta V = (2\pi r)(h)\,\Delta x$

$= 2\pi(x+1)\left(2x - \frac{1}{2}x^2\right)\Delta x$

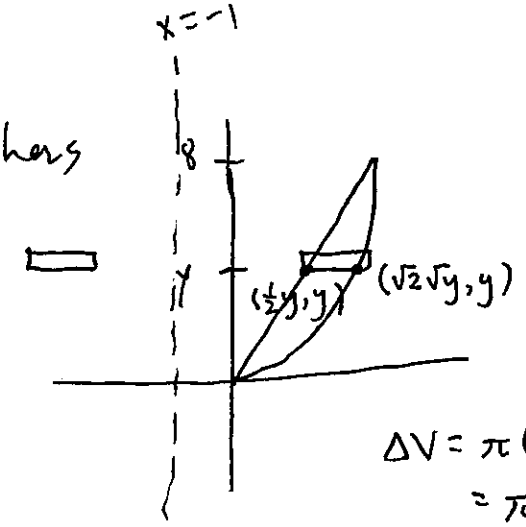
$V = \int_0^4 2\pi(x+1)\left(2x - \frac{1}{2}x^2\right)dx$

$= 2\pi \int_0^4 2x^2 + 2x - \frac{1}{2}x^3 + \frac{1}{2}x^2\,dx$

$= 2\pi \int_0^4 \frac{3}{2}x^2 + 2x - \frac{1}{2}x^3\,dx$

$= 2\pi \left[\frac{1}{2}x^3 + x^2 - \frac{1}{8}x^4\right]_0^4$

$= 2\pi\left[32 + 16 - 32\right] = \boxed{32\pi}$



$y = \frac{1}{2}x^2$

$2y = x^2$

$x = \sqrt{2y}$ if $x > 0$

$y = 2x$

$x = \frac{1}{2}y$

$$\Delta V = \pi (r_2^2 - r_1^2)\Delta y$$

$$= \pi\left((\sqrt{2}\sqrt{y}+1)^2 - (\tfrac{1}{2}y+1)^2\right)\Delta y$$

$$= \pi\left[2y + 2\sqrt{2}\sqrt{y} + 1 - (\tfrac{1}{4}y^2 + y + 1)\right]\Delta y$$

$$= \pi\left[y + 2\sqrt{2}\sqrt{y} - \tfrac{1}{4}y^2\right]\Delta y$$

$$V = \int_0^8 \pi\left[y + 2\sqrt{2}\, y^{1/2} - \tfrac{1}{4}y^2\right] dy$$

$$= \pi\left[\frac{y^2}{2} + 2\sqrt{2}\cdot\frac{2}{3}y^{3/2} - \frac{1}{12}y^3\right]_0^8$$

$$= \pi\left[32 + \frac{4\sqrt{2}}{3}(2\sqrt{2})^3 - \frac{1}{12}\cdot 8\cdot 8\cdot 8\right]$$

$\sqrt{8} = 2\sqrt{2}$

$$= \pi\left[32 + \frac{4}{3}\cdot 32 - \frac{1}{3}\cdot 128\right]$$

$$\boxed{= 32\pi}$$