

Math 1210-001
Friday Feb 26
WEB L112

• 3.2-3.3 Finish Wednesday's notes about the section 3.2 topics of monotonicity and concavity. Then continue into today's notes which introduce the section 3.3 topics of local extrema, and extrema on open intervals.

Definitions: Let f have domain S , with c in S .

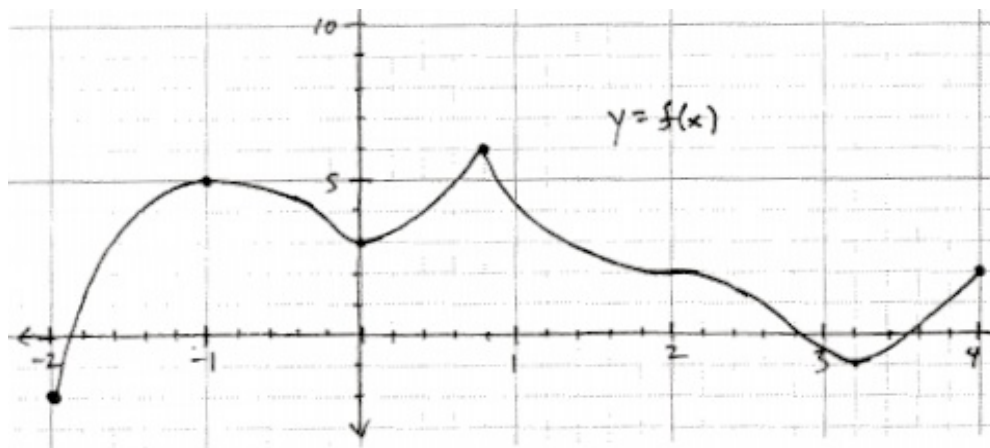
(i) $f(c)$ is a local maximum value of f if there is an open interval (a, b) with $a < c < b$ so that $f(c)$ is the maximum value of f on the intersection of (a, b) with S

(ii) $f(c)$ is a local minimum value of f if there is an open interval (a, b) with $a < c < b$ so that $f(c)$ is the minimum value of f on the intersection of (a, b) with S .

(iii) $f(c)$ is a local extreme value of f if $f(c)$ is a local minimum value or a local maximum value of f .

Remark Local extreme values can only occur at critical points, since they are extreme values for subintervals of the original domain.

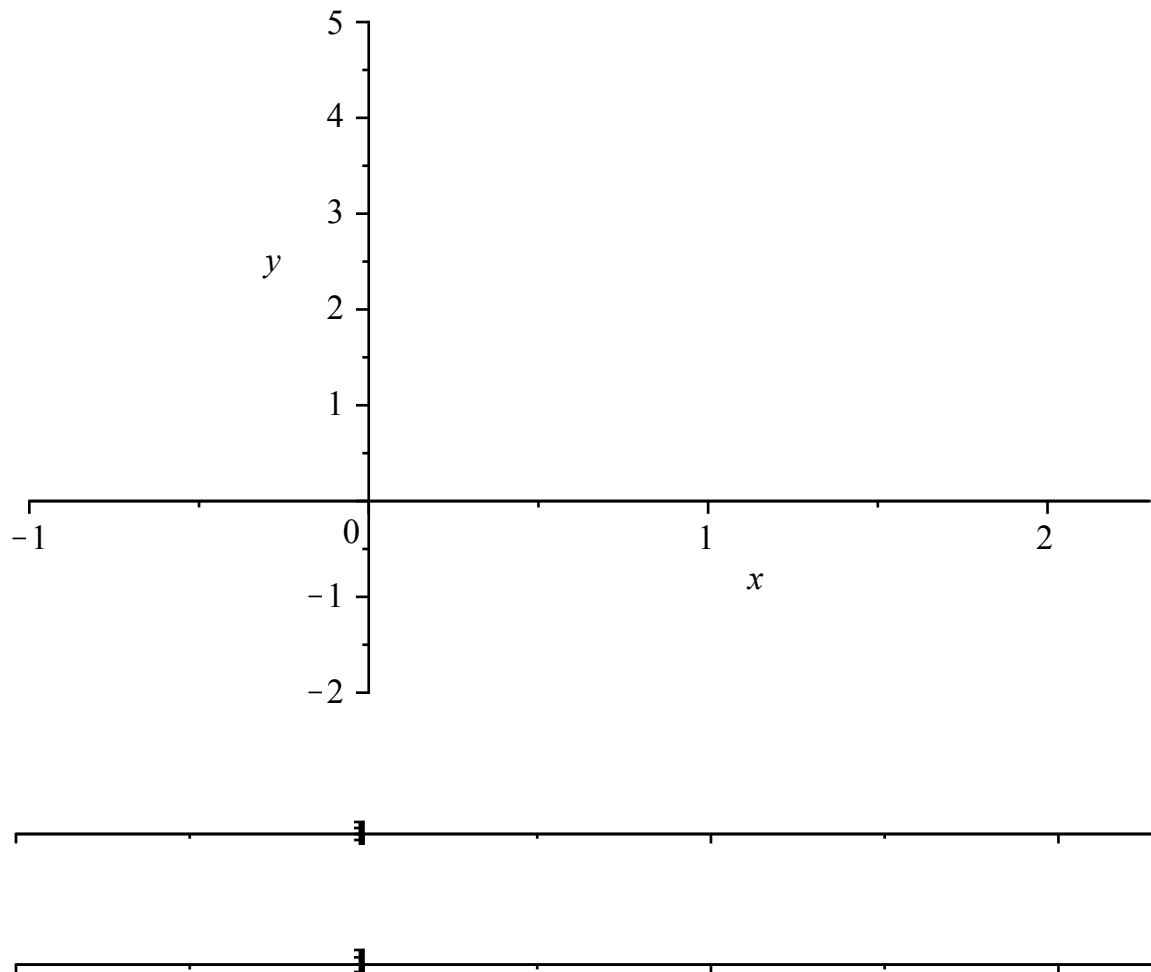
Exercise 1) Identify the critical points and also the local extreme values, for the function f on the domain $[-2, 4]$, whose graph is shown below.



Exercise 2) Use first and second derivative information to sketch the graph of

$$y = x^4 - 2x^3.$$

(Assume the domain is all real numbers, even though you are only sketching a portion of the graph which has been pre-selected.) Indicate all intercepts, critical points, local extrema. On the line segments below the graph indicate the largest intervals on which the function is increasing/decreasing, and concave up/concave down.

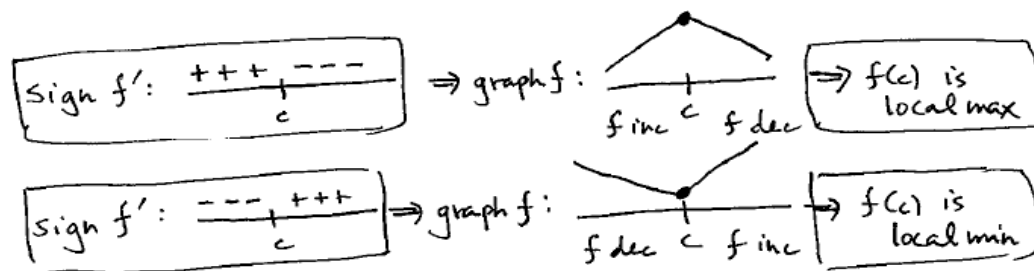


There are first and second derivative tests that one can apply at critical points, to see whether they are the locations of local maxima or local minima. We've implicitly been using the first derivative test in many of our examples already. Both tests make sense, and are explainable:

First derivative test: Let $f(x)$ be continuous in an open interval $I = (a, b)$ containing c , and differentiable in that interval except maybe at c . Then

- (i) If $f'(x) < 0$ for $a < x < c$ and $f'(x) > 0$ for $c < x < b$ then $f(c)$ is a local minimum.
- (ii) If $f'(x) > 0$ for $a < x < c$ and $f'(x) < 0$ for $c < x < b$ then $f(c)$ is a local maximum.

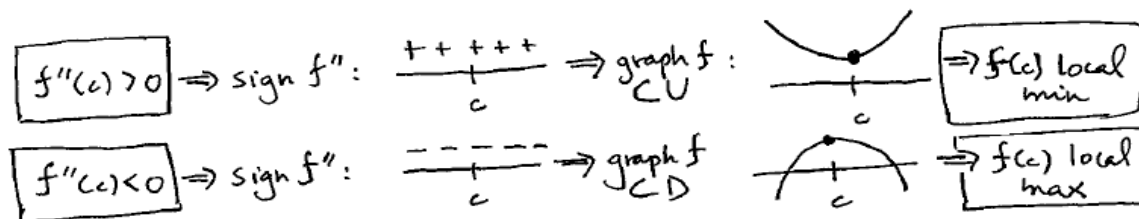
Pictorially,



Second derivative test: Let $f(x)$ be differentiable on an open interval I containing c and let c be a stationary point ($f'(c) = 0$). If f'' is continuous on I and

- (i) $f''(c) > 0$ then $f(c)$ is a local minimum.
- (ii) $f''(c) < 0$ then $f(c)$ is a local maximum.

Pictorially,



Exercise 3) Check how the first and second derivative tests apply for the stationary points in Exercise 2 above. If time, also revisit the two examples from Wednesday's notes.