

Math 1210-001
Friday Feb 19
WEB L112

2.8 Related rates, continued.

In section 2.7 we used identity equations and the chain rule to find the derivatives of functions at given points on their graphs, without needing to know the explicit formulas for the functions. This method is called implicit differentiation. In section 2.8 we use identities involving several different functions (of time) at once, along with the chain rule, in order to figure out how fast one of those functions is changing with respect to time, in terms of how fast the others are. This topic is called related rates.

- Do Exercise 3 from Wednesday's notes; talk a bit about Exercise 4 which is related to one of your lab problems. Then discuss examples from today's notes. Related rates problems can be approached systematically:

steps:

- 1) Construct a diagram with the relevant functions (and constants) labeled.
- 2) What is known? What is asked for? Express quantities mathematically as function values and/or derivatives.
- 3) Relate the known and unknown functions with one or more identity equations.
- 4) Differentiate the identity equation in 3, with respect to time. Most likely this will require use of the chain rule, like in implicit differentiation problems.
- 5) Using the identity equations and their differentiated versions, use the known function and derivative values to find the unknown one.

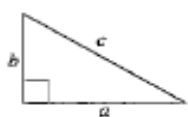
Exercise 1 (related to lab 5 #5). Sand is pouring from a pipe at a rate of $16 \frac{ft^3}{sec}$. If the falling sand forms a conical pile on the ground whose altitude is always $\frac{1}{4}$ the diameter of the base, how fast is the altitude increasing when the pile is 4 ft. high? (Hint: the volume of a right-circular cylinder of radius r and height h is given by $V = \frac{1}{3} \pi r^2 h$.)

Exercise 2) (#15, page 141). A light in a lighthouse 1 km off-shore is rotating at 2 revolutions per minute. How fast is the beam moving along the shoreline when it passes the point $\frac{1}{2}$ km from the point opposite the lighthouse?

Exercise 3) An airplane flying west at $300 \frac{\text{miles}}{\text{hour}}$ goes over the control tower at noon, and a second airplane at the same altitude, flying north at $400 \frac{\text{miles}}{\text{hour}}$, goes over the tower an hour later. How fast is the distance between the two airplanes changing at 2:00 p.m.?

Calculus class related rates problems often use the following geometry facts, as well as similar triangles and basic trigonometry facts. This is not because its these identities that are most likely to arise in real-world related rates problems; rather it's because these identities are assumed to be in calculus student toolboxes.

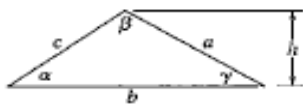
GEOMETRY



Right triangle


Triangles

Pythagorean Theorem
 $a^2 + b^2 = c^2$




Any triangle

Angles $\alpha + \beta + \gamma = 180^\circ$
 Area $A = \frac{1}{2}bh$




Circles

Circumference $C = 2\pi r$
 Area $A = \pi r^2$




Cylinders

Surface area $S = 2\pi r^2 + 2\pi rh$
 Volume $V = \pi r^2 h$



Cones

Surface area $S = \pi r^2 + \pi r\sqrt{r^2 + h^2}$
 Volume $V = \frac{1}{3}\pi r^2 h$



Spheres

Surface area $S = 4\pi r^2$
 Volume $V = \frac{4}{3}\pi r^3$

CONVERSIONS

1 inch = 2.54 centimeters	1 kilometer \approx 0.62 miles
1 liter = 1000 cubic centimeters	1 liter \approx 1.057 quarts
1 kilogram \approx 2.20 pounds	1 pound \approx 453.6 grams
π radians = 180 degrees	1 cubic foot \approx 7.48 gallons