Name Solutions

UID

Math 1210-3

Quiz 9

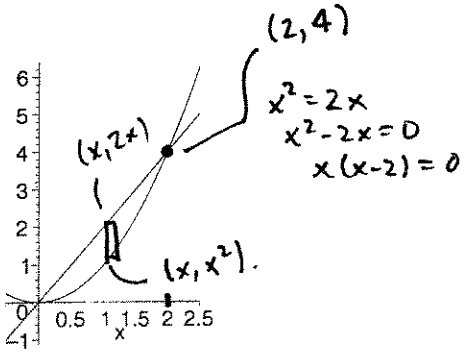
April 18, 2008

Show all work for complete credit!

1) Consider the region bounded by the graphs of $y = x^2$ and $y = 2x$.

1a) Find the area of this region. (The region is sketched below.)

(2 points)

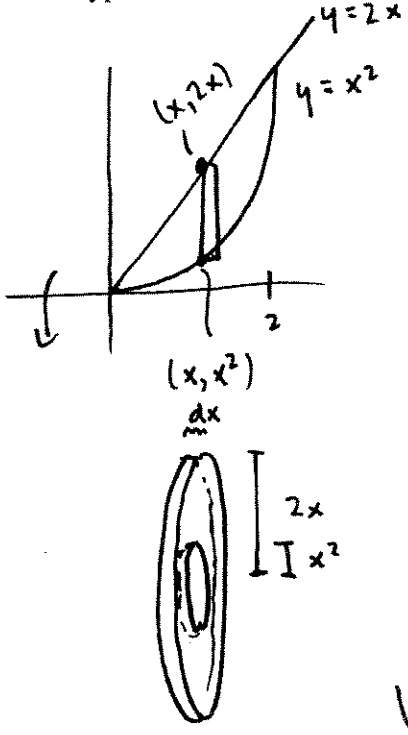


$$dA = (2x - x^2)\,dx$$

$$A = \int_0^2 2x - x^2\,dx = x^2 - \frac{x^3}{3}\Big]_0^2 = 4 - 8/3 = \boxed{4/3}$$

1b) Consider the object created by rotating the region above about the x-axis. Compute its volume using planar slabs (which are washers in this case). You may want to redraw the sketch above, and draw and label a typical washer, to help set up the integral.

(4 points)



$$dV = \pi(r_2^2 - r_1^2)\,dx$$

$$= \pi(4x^2 - x^4)\,dx$$

$$V = \int_0^2 \pi(4x^2 - x^4)\,dx = \pi\int_0^2 4x^2 - x^4\,dx$$

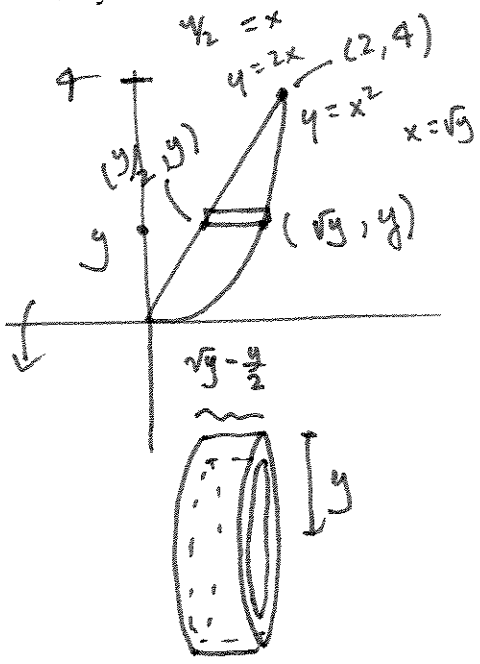
$$\pi\left[\frac{4}{3}x^3 - \frac{x^5}{5}\right]_0^2$$

$$\pi\left[\frac{32}{3} - \frac{32}{5}\right]$$

$$32\pi\left(\frac{1}{3} - \frac{1}{5}\right)$$

$$32\pi\left(\frac{2}{15}\right) = \boxed{\frac{64\pi}{15}}$$

1c) Recompute the the volume in part (1b), but using cylindrical shells rather than planar slabs. You may want to redraw the region, and draw and label a typical cylindrical shell, to help set up the integral. (4 points)



$$dV = (2\pi y)\left(\sqrt{y} - \frac{y}{2}\right) dy$$

$$= 2\pi \left(y^{3/2} - \frac{y^2}{2}\right) dy$$

$$V = \int_0^4 2\pi \left(y^{3/2} - \frac{y^2}{2}\right) dy = 2\pi \left[\frac{2}{5} y^{5/2} - \frac{y^3}{6}\right]_0^4$$

$$= 2\pi \left[\frac{2}{5} \cdot 2^5 - \frac{64}{6}\right]$$

$$= 2\pi \left[\frac{64}{5} - \frac{64}{6}\right]$$

$$= 128\pi \left(\frac{1}{30}\right)$$

$$= \frac{64\pi}{15}$$