Name SOLUTIONS

UID

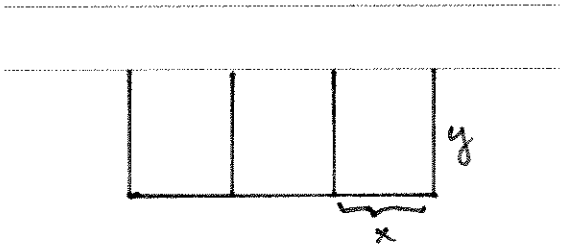
Math 1210-3

Quiz 7

March 14, 2008

Show all work for complete credit!

1) Farmer Joe wishes to test three new strains of table corn, as well as advertise his farming abilities. He will fence off three adjacent and congruent rectangular plots, one for each variety of corn, using the road as the "northern" (fenceless) border for each plot. The road edge is represented with the dotted lines in the picture below, and the fencing is represented with solid line segements.



Farmer Joe happens to have 600 linear feet of fencing available to construct his plots. What dimensions should he choose for each rectangular plot in order to maximize its area?

1a) Find the answer to the question above using Calculus.

(7 points)

Maximize $A = 3xy$

subject to constraint $600 = 4y + 3x$

$4y = 600 - 3x$

$y = 150 - \frac{3}{4}x$

$A(x) = 3x\left(150 - \frac{3}{4}x\right) = 450x - \frac{9}{4}x^2 \qquad 0 \le x \le 600$

stat pt: $A'(x) = 450 - \frac{9}{2}x = 0$

$\frac{9}{2}x = 450$

$9x = 900$

$x = 100$ ft.

$x = 100$ ft

$y = 150 - 75 = 75$ ft

MAXIMUM area. (typo!)

1b) Explain logically and mathematically why your answer gives ~~MINIMUM cost~~, using concepts we've been discussing. (Use the back of the page if necessary). Precisely, use critical point reasoning on an appropriate domain for your function, and/or an INC/DEC or CU/CD argument.

(3 points)

Crit pt theory: for $0 \le x \le 600$ $A(x) \ge 0$. $A(0) = A(600) = 0$

so max value must occur at interior critical pt. There is only one of these, @ $x = 100$, so $A(100)$ must be the max value.

or

CU/CD: $A''(x) = -\frac{9}{2} < 0$ so graph of A is CD on entire interval, and the stationary pt must be the location of a maximum value

or

INC/DEC: $A'(x) = 450 - \frac{9}{2}x$: sign A': $\underset{\text{INC } 100 \text{ DEC}}{+\,+\,+\;0\;-\,-\,-} \Rightarrow A(100) =$ Max value!