Name.....

UID.....

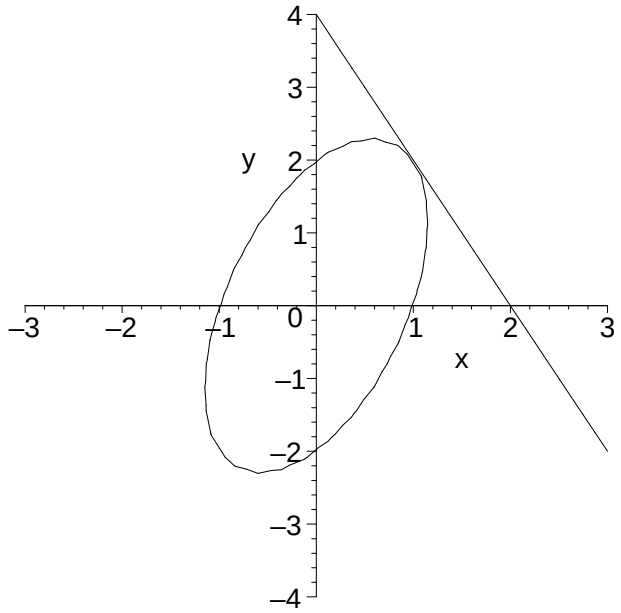
Math 1210-3

Quiz 6 Solutions

February 22, 2008

Show all work for complete credit! There are two sides to this quiz!

This quiz is about the graph of the equation $4x^2 - 2xy + y^2 = 4$. Here is a computer picture of the graph: *(The tangent line has been drawn in.)*



1a) Use algebra to show that the point (1, 2) is on the graph shown above.

(2 points)

To be on the graph means precisely that (x,y) satisfies the equation, and 4(1)-2(2)+4 =4 is true, so the point (1,2) is on the graph.

1b) Use implicit differentiation to find the slope of this equation graph, at the point (1, 2) .

(4 points)

We treat "y" as a function "y(x)". Then the equation says two expressions in x are equal, so their x-derivatives must also be .... this is the principle of implicit differentiation. Using the product rule and the chain rule we get

$$8x - 2y - 2xy' + 2yy' = 0$$

Collecting y' terms on one side of this equation, and the rest on the other side, and factoring out the y' yields

$$y'(-2x+2y) = 2y - 8x$$

i.e. (after first dividing both sides of the equation by 2, and then solving for y'),

$$y'=(y-4x)/(-x+y)$$

At the point x=1, y=2 this yields

$$y'=-2.$$

This is the slope of the graph at (1,2).

1c) Find the slope-intercept equation of the tangent line at the point (1, 2), for the graph shown on the previous page.

(2 points)

Use point-slope form, then convert:

$$y-2=-2(x-1)$$
$$y=-2x+4$$

1d) Sketch the tangent line you found in (1c), onto the graph at the top of the first page. Make sure it has the correct slope and x and y intercepts.

(2 points)

The slope is negative two. The y-intercept is 4. The x-intercept (when y=0) is 2. The line is included in the page 1 picture.