Name..
UID...

Math 1210-3
Quiz 5 Solutions
February 15, 2008

Show all work for complete credit. There are two sides to this quiz!

Compute the following derivatives. Do not simplify your answers unless the question asks you to.

1) Find $D_x\left(x^3 + \frac{5}{x^2}\right)(4x+5)$

(2 points)

Using the product rule:

$$\frac{d}{dx}\left(\left(x^3+\frac{5}{x^2}\right)(4x+5)\right) = \left[3x^2 - \frac{10}{x^3}\right](4x+5) + \left(x^3+\frac{5}{x^2}\right)[4]$$

Alternately, you could "foil" the function first, and then use polynomial calculus to get an answer agreeing with your product rule computation.

2) Use the quotient rule to find $D_x\left[\frac{\cos(x)}{\sin(x)}\right]$, and simplify your answer to show that your computation reproduces the derivative formula for $\cot(x)$ which you have memorized.

(2 points)

$$\frac{d}{dx}\left(\frac{\cos(x)}{\sin(x)}\right) = \frac{-\sin(x)^2 - \cos(x)^2}{\sin(x)^2}$$

$$= -\frac{1}{\sin(x)^2}$$

(we just used the Pythagorean trig identity),

$$=-\cot(x)^2.$$

3) Find $D_x\sqrt{x^2+1}$.

(2 points)

Use the chain rule:

$$D_x\sqrt{x^2+1} = D_x\left(x^2+1\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(x^2+1\right)^{-\frac{1}{2}} 2x$$

$$= \frac{x}{\sqrt{x^2+1}} .$$

4) Find f '(t), for $f(t) = \left(t^2 + 3t + 4\right)^{11} \left(t^3 + \frac{1}{t^3}\right)^9$.

(2 points)

Use the product rule, and then the chain rule:

$$f'(t) = 11\left(t^2 + 3t + 4\right)^{10} (2t + 3)\left(t^3 + \frac{1}{t^3}\right)^9 + \left(t^2 + 3t + 4\right)^{11} 9\left(t^3 + \frac{1}{t^3}\right)^8 \left(3t^2 - \frac{3}{t^4}\right)$$

5) Find $D_t\left[\frac{\left(t^2 + 1\right)^{11}}{\sec(2t)}\right]$.

(2 points)

Use the quotient rule, and then the chain rule:

$$D_t\left[\frac{\left(t^2 + 1\right)^{11}}{\sec(2t)}\right] = \frac{11\left(t^2 + 1\right)^{10} 2t\,\sec(2t) - \left(t^2 + 1\right)^{11}(\sec(2t)\tan(2t))2}{\sec(2t)^2}.$$

The more clever way to simplify this problem is to use the fact that sec and cos are multiplicative reciprocals, so

$$D_t\left[\frac{\left(t^2 + 1\right)^{11}}{\sec(2t)}\right] = D_t\left(t^2 + 1\right)^{11}\cos(2t)$$

$$= 11\left(t^2 + 1\right)^{10} 2t\cos(2t) - \left(t^2 + 1\right)^{11}(\sin(2t))2.$$

(You should be able to see that these two answers agree!)