Name.....

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Math 1210-3

Quiz 4 Solutions

February 8, 2008

Show all work for complete credit! There are two sides to this quiz!

1a) Find which step is wrong in the following computation, and explain why it was wrong:

$$\lim_{x\to 0} x\cot(3x) = \lim_{x\to 0} \frac{x\cos(3x)}{\sin(3x)} = \lim_{x\to 0} \frac{3x\cos(x)}{3\sin(x)} = \left(\lim_{x\to 0} \frac{x}{\sin(x)}\right)(\lim_{x\to 0} \cos(x)) = 1.$$

(2 points)

Equating the expressions is the second equality is the bad step: It is not true that $\cos(3x) = 3\cos(x)$, or that $\sin(3x) = 3\sin(x)$. *All the other equalities are valid.*

1b) Compute $\lim_{x\to 0} x\cot(3x)$ correctly.

(3 points)

$$\lim_{x\to 0} x\cot(3x) = \lim_{x\to 0} \frac{x\cos(3x)}{\sin(3x)} = \left(\lim_{x\to 0} \frac{1}{3}\right)\left(\lim_{x\to 0} \frac{3x}{\sin(3x)}\right)(\lim_{x\to 0} \cos(3x)).$$

Since $\lim_{h\to 0} \frac{h}{\sin(h)} = 1$, let $h = 3x$, *and see that middle limit equals 1. Thus our limit is the product of* $\frac{1}{3}$,1, *and* 1, *i.e.* $\frac{1}{3}$.

2a) What value of a makes the function $f(x)$ continuous at $x = 1$?

$$f(x) = 2\,x + a, \text{ for } x \le 1;$$
$$f(x) = 2\,a\,x^2, \text{ for } 1 < x\,.$$

(3 points)

We need

$$\lim_{x \to 1-} f(x) = \lim_{x \to 1+} f(x)$$

(and that these two limits also equal $f(1)$*. Evaluating these two limits yields*

$$2 + a = 2\,a$$

which has solution

$$a = 2\,.$$

2b) Sketch the continuous function you found in (3a).

(2 points)

We are sketching the function defined by $f(x) = 2\,x + 2$, *for* $x \le 1$, *and* $f(x) = 4\,x^2$, *for* $1 \le x$.