Name.....

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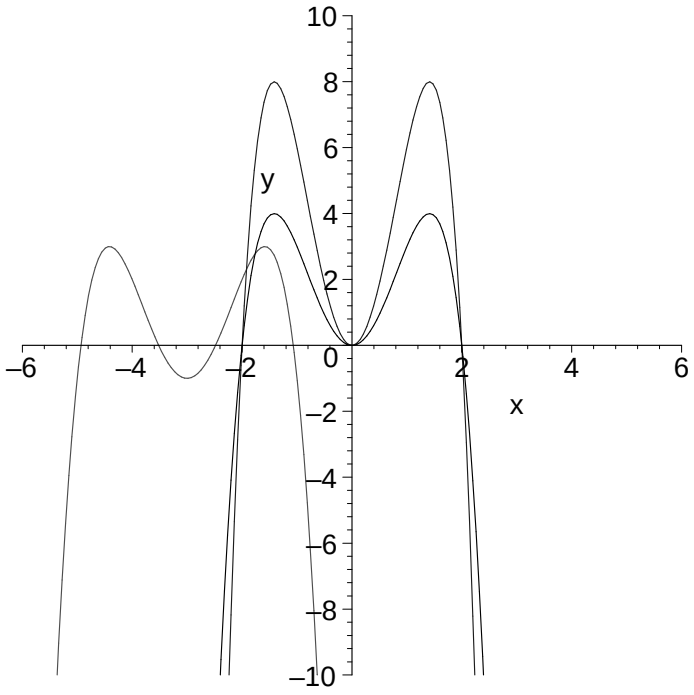
Math 1210-3

Quiz 3 Solutions

January 25, 2008

Show all work for complete credit! There are two sides to this quiz!

Here is a computer sketch of the graph $y = f(x)$, for the polynomial function $f(x) = -x^4 + 4x^2$. All the problems on the quiz are related to this particular function and its graph.



1a) How is the graph of $y = -2x^4 + 8x^2$ related to the graph of $y = f(x)$ shown above? Answer this question in words, and then carefully sketch the graph of $y = -2x^4 + 8x^2$ into the picture above.

(2 points)

This is the graph of $y = 2f(x)$, so its graph is the original graph, stretched vertically by a factor of 2. (For a given x-coordinate, the new y-coordinate is twice the original one.) The graph is shown above - it's the skinny one which still goes through the origin.

1b) How is the graph of $y = -(x+3)^4 + 4(x+3)^2 - 1$ related to the graph of $y = -x^4 + 4x^2$? Answer this question in words, and then carefully sketch the new graph into the picture above.

(2 points)

This is the graph of $y = f(x+3) - 1$, so the original graph has been shifted horizontally to the left (!!) by 3 units, and vertically down by 1 unit. For example, the "vertex" (-3,-1) is on the new graph, and corresponds to the original "vertex" (0,0).

1c) Is the function $f(x) = -x^4 + 4x^2$ even, odd, or neither? Explain your answer, using the definition of what even or odd means. How is this even or odd property reflected in a symmetry property of the graph $y = -x^4 + 4x^2$?

(2 points)

Because f(x) has only even powers, we compute

$$f(-x) = -[-x]^4 + 4[-x]^2 = -x^4 + 4x^2 = f(x)$$

and this verifies the definition that f(x) be an even function. Even functions have graphs which are symmetric with respect to the y-axis (because if (x,f(x)) is on the graph, then so is the reflected point (-x,f(-x))=(-x,f(x))). And this is shown in the page 1 picture.

1d) Let $g(x) = \sqrt{x}$, and $f(x) = -x^4 + 4x^2$ as always. What is the formula for $g\,f$? What is the natural domain of this new function?

(2 points)

The product function has formula $gf(x) = \sqrt{x}\,(-x^4 + 4x^2)$*, and so has natural domain* $0 \le x$ *, since precisely the non-negative numbers have real square root.*

1e) Using the same functions as in part (1d), What is the formula for the function $g\ o\ f$? . What is the natural domain for this new function? (Hint: Let the graph on page one help you.)

(2 points)

The composition function

$$g\ o\ f(x) = g(-x^4 + 4x^2) = \sqrt{-x^4 + 4x^2},$$

and the natural domain is those x-values for which $-x^4 + 4x^2$ *is non-negative. The graph on page 1 indicates that these are exactly those x's for which*

$$-2 \le x \le 2, \quad \text{i.e.} \quad |x| \le 2$$

In fact, we can show this algebraically too, since $-x^4 + 4x^2 = x^2(-x^2 + 4)$*, and this expression is greater than or equal to zero exactly when* $x^2 \le 4$, i.e. $|x| \le 2$.