Name.....

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Math 1210-3

Quiz 2

January 18, 2008

Show all work for complete credit! Every question below relates to the function $f(x) = -3x^2 + 12x$. There are questions on both sides of the paper!

1a) Compute the derivative $f'(x)$.

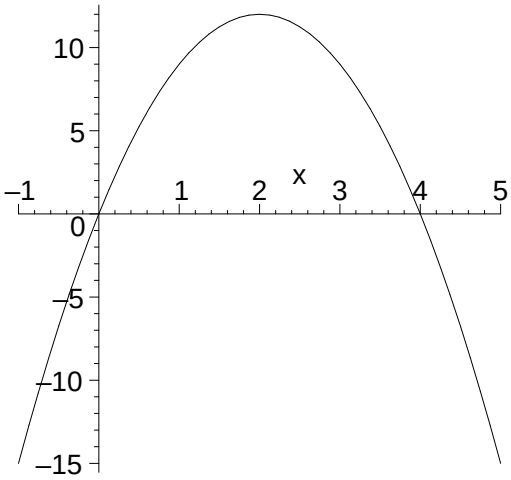
(1 point)

$$f'(x) = -6x + 12 .$$

1b) What is the value of $f'(0)$ and what does it have to do with the graph $y = -3x^2 + 12x$ sketched below?

(2 points)

$f'(0) = 12$. This is the slope of the graph below, at the point (0,0). If you were to draw in the tangent line and measure its slope using the horizontal and vertical scales, you should get rise/run close to 12. (See quiz 1.)



1c) Compute $\int -3x^2 + 12x\,dx$.

(2 points)

$$\int -3x^2 + 12x\,dx = -x^3 + 6x^2 + C$$

1d) Compute $\int_0^4 -3x^2 + 12x\,dx$.

(1 point)

Using the antiderivative $F(x) = -x^3 + 6x^2$, *and the Fundamental Theorem of Calculus we compute*

$$\int_0^4 -3x^2 + 12x\,dx = F(4) - F(0) = -64 + 96 - 0 = 32 .$$

1e) What does the number you just computed in (1d) have to do with the graph $y = -3x^2 + 12x$ sketched above?

(1 point)

The number 32 is the area of the region between the curve $y = -3x^2 + 12x$ *and the x-axis, for x between 0 and 4. Notice that if you draw a rectangle in the picture above, whose base is the x-interval from 0 to 4, and whose height is y=8, then this rectangle has the same area as the region, and even looks like it does.*

1f) An object is moving along a number line with velocity $v = -3t^2 + 12t$ feet per second, at time t seconds. What is its acceleration at time $t = 0$? Include correct units!

(1 point)

$$a = D_t(-3t^2 + 12t) = -6t + 12 .$$

At t=0, the value of the acceleration is $12\ \frac{ft}{\sec^2}$. *Notice, we did this computation using different letters and units, in (1a).*

1g) If the object in (1f) is at position $s(0) = 3$ feet initially, then where is located when $t = 4$ seconds ?

(2 points)

Using Fundamental Theorem of Calculus,

$$\int_0^4 -3t^2 + 12t\,dt = s(4) - s(0).$$

But we computed the value of this integral in (1d), and it is 32 (feet). Therefore

$$s(4) - s(0) = 32$$
$$s(4) = 32 + s(0)$$
$$s(4) = 35 \text{ feet.}$$

Alternately, you could do this problem by just using antidifferentiation:

$$s = \int -3t^2 + 12t\,dt.$$
$$s = -t^3 + 6t^2 + C.$$

Since $s(0)=3$, *deduce* $C = 3$, *so*

$$s = -t^3 + 6t^2 + 3$$

and when $t = 4$, $s = 35$ *feet.*