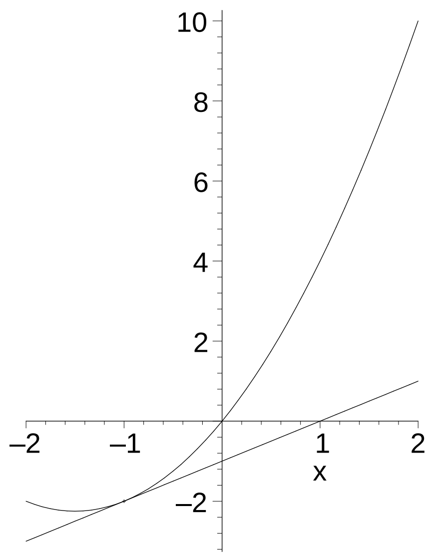
Math 1210-3
Quiz 1 Solutions
January 11, 2008

1a) Using the graph of $y = x^2 + 3\,x$ shown below, label the point (-1,-2) on the graph and then draw the tangent line to the graph, through the point (-1,-2), i.e. the line which looks like it has the same slope as the graph does at (-1,-2), and which passes through that point. You may wish to fold over edges of your paper to use as straightedges, in order to accurately locate (-1,-2), and then to draw the tangent line.

(2 points)



1b) Use the limit definition of derivative to compute the slope function f′(x), for $f(x) = x^2 + 3\,x$. In other words, first compute the secant line slopes $\frac{f(x+h) - f(x)}{h}$, and then use algebra to work out what value they approach as h approaches zero.

(3 points)

$$f(x+h) = (x+h)^2 + 3\,(x+h) \;\; = x^2 + 2\,h\,x + h^2 + 3\,x + 3\,h\,.$$
$$f(x) = x^2 + 3\,x$$

So, subtracting and then cancelling terms,

$$f(x+h) - f(x) = 2\,h\,x + h^2 + 3\,h$$
$$\frac{f(x+h) - f(x)}{h} = 2\,x + h + 3$$

and $f'(x)$ is given by

$$\lim_{h \to 0}\; 2\,x + h + 3 = 2\,x + 3.$$

1c) Use your answer from (1b), or the differentiation rules we've learned for polynomials in case you don't trust that answer, to deduce that the graph of $y = x^2 + 3x$ has slope m=1 when x=-1.

(2 points)

Since $f'(x) = 2x + 3$, *the slope at x = 1 is*

$$f'(-1) = 2(-1) + 3 = 1.$$

1d) Find the slope-intercept equation of the tangent line you drew in part (1a). You can check whether your equation is likely correct by comparing it to the line you sketched.

(3 points)

The line goes through (-1,-2) and has slope equal to 1, so the point-slope equation is

$$y + 2 = 1(x + 1).$$

This yields slope-intercept form

$$y = x - 1.$$