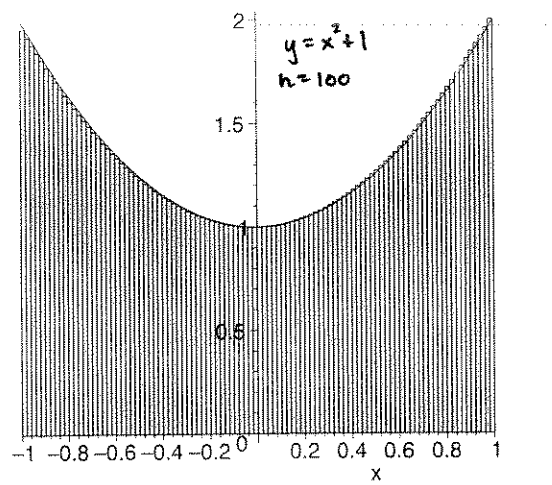
Math 1210-3
Monday March 31.

§ 4.1-4.2 The definite integral.

Recall (and review), on Friday we defined the definite integral as a limit of Riemann sums. This limit always exists if $f(x)$ is continuous on $[a,b]$, and one of its interpretations is that it gives the signed area between the graph of f and the x-axis.

We will work two exercises carefully, which show how to set up the Riemann sums and take the limit. Along the way we'll practice using summation notation.

0. Work Exercise 1 on Friday's notes



1. Work the following example

$$\int_{-1}^{1} x^2+1 \ dx$$

partition into n equal length subintervals.
Deduce their widths Δx.
Find the i^{th} subinterval I_i
Use sample point $\bar{x}_i$ = right-endpoint.
Express R_p.
Simplify R_p with magic formulas, take limit as $n \to \infty$, to get the value of the definite integral.

General case: Subdivide $[a,b]$ into n equal-length subintervals of width Δx. Then

$\Delta x =$

$I_i =$

(Part II of) The Fundamental Theorem of Calculus relates antidifferentiation to the definite integral. It says that if $f(x)$ has an antiderivative $F(x)$ on $[a,b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Note: we abbreviate $F(b) - F(a)$ as $F(x)\Big]_a^b$.

Exercise 2: Check the Friday example, and the one we just worked, with the FTC. Isn't it amazing??

2a) $\displaystyle\int_0^3 x+2\,dx =$

2b) $\displaystyle\int_{-1}^1 x^2+1\,dx =$

Exercise 3 Since $\int_a^b f(x)\,dx$ gives signed area as one interpretation,

Compare what geometry says the answer is, to what the FTC says the answer is, in the following examples. Sketch first!

3a) $\int_0^2 5\,dx$

3b) $\int_{-1}^2 3x\,dx$

3c) $\int_{-2}^2 2x+6\,dx$

3d) $\int_{-2}^2 \sqrt{4-x^2}\,dx$

(geometry works best on this one!!)