Math 1210-3

Monday March 4 §3.1-3.3

Exam comments: Overall, your scores on #1-3 were respectable. (That means "good")

But, #4 related rates

#5 differentials

gave a lot of people serious trouble! (See score distributions in today's notes.)

So, instead of a quiz this Friday you will have a chance to make up either or both your individual #4, #5 scores - there will be a related rates problem worth 25 pts, and a differentials problem worth 10 points. You'll have the last 25 minutes of class. (We recorded your exam score as well as what each of you got on #4,5.) If either or both your scores on #4,5 improve, you get to keep the new score, and we'll recalculate your exam total.

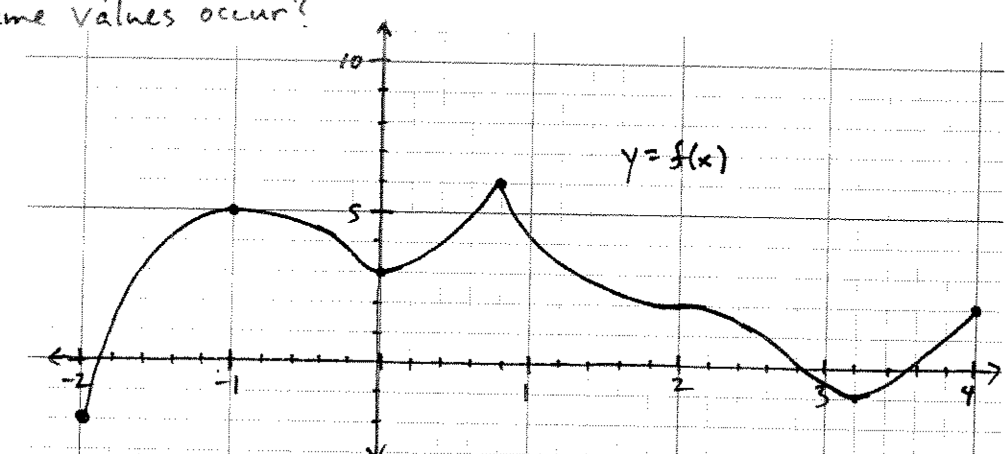
Last Tuesday - Wednesday we were discussing max/min issues.

Exercise 1: Recall our discussion.

1a) What is an extreme value for a function f, defined on a domain S?

1b) If f is defined on an interval, what are the three possible locations c at which $f(c)$ can have an extreme value? What are such points called?

1c) Here is a graph $y = f(x)$, of a continuous function on the interval $[-2,4]$ (so we know f attains its maximum and minimum value.). Identify each critical point, and classify it. At which critical points do the extreme values occur?



Sometimes we care about <u>local extrema</u> (as opposed to global extrema)

<u>Definition</u>:

Let f have domain S, with c an element of S.

Then

(i) $f(c)$ is a <u>local maximum value</u> of f if there is an interval $(\tilde{a},\tilde{b})$ containing c, so that $f(c)$ is the maximum value of f on $(\tilde{a},\tilde{b}) \cap S$.

intersection

(ii) $f(c)$ is a <u>local minimum value</u> of f if there is an interval $(\tilde{a},\tilde{b})$ containing c, so that $f(c)$ is the minimum value of f on $(\tilde{a},\tilde{b}) \cap S$.

(iii) $f(c)$ is a <u>local extreme value</u> of f if it is either a local max. value or a local min. value.

<u>Exercise 2</u> For the function f on page 1, identify all local extreme values, as well as where they occur.

<u>Notice</u>: Every local extreme value <u>must</u> occur at a critical point (the same explanation for why global extreme values occur at critical points works for local extreme points.)

<u>But</u>, critical points do not have to be the location of local extrema (e.g. @ $x=2$ in page 1 example).

<u>Exercise 3</u> Can you think of a function $f(x)$ with $f(0)=0$, $f'(0)=0$ but $f(0)$ is neither a local max nor a local min?

More language!

<u>Definition</u> Let f be defined on an interval I (open, closed, or neither.)

We say

(i) f is <u>increasing</u> on I iff whenever $x_1 < x_2$ then $f(x_1) < f(x_2)$

(graph rises to the right.)

(ii) f is <u>decreasing</u> on I iff whenever $x_1 < x_2$, then $f(x_1) > f(x_2)$.

(iii) f is <u>strictly monotone</u> on I

iff it is either increasing on I, or decreasing on I

<u>Exercise 4</u> Identify the largest intervals on which the page 1 f is strictly monotone.

<u>Exercise 5</u> What are (is?) the largest interval(s?) on which $f(x) = x^3$ is strictly monotone?

This result is intuitively sensible – we'll prove it later in the chapter:

<u>Theorem A</u> : If f is continuous on I and differentiable at each interior point of I (not an endpoint)

then

(i) $f'(x) > 0$ $\forall$ interior pts $x \Rightarrow f$ increasing on I

(ii) $f'(x) < 0$ $\forall$ interior pts $x \Rightarrow f$ decreasing on I

<u>Definition</u> : f is <u>concave up</u> on I if (the slope) f' is increasing on I 

f is <u>concave down</u> on I if f' is decreasing on I 

applying Theorem A to $f'(x)$ we deduce

<u>Theorem B</u> Let f be twice differentiable on I. Then

(i) $f''(x) > 0$ $\forall$ interior points $\Rightarrow f$ is concave up (CU) on I

(ii) $f''(x) < 0$ " " $\Rightarrow$ " concave down (CD) on I

<u>Exercise 6</u> : Use first and second derivative information to graph $y = x^4 - 2x^3$.

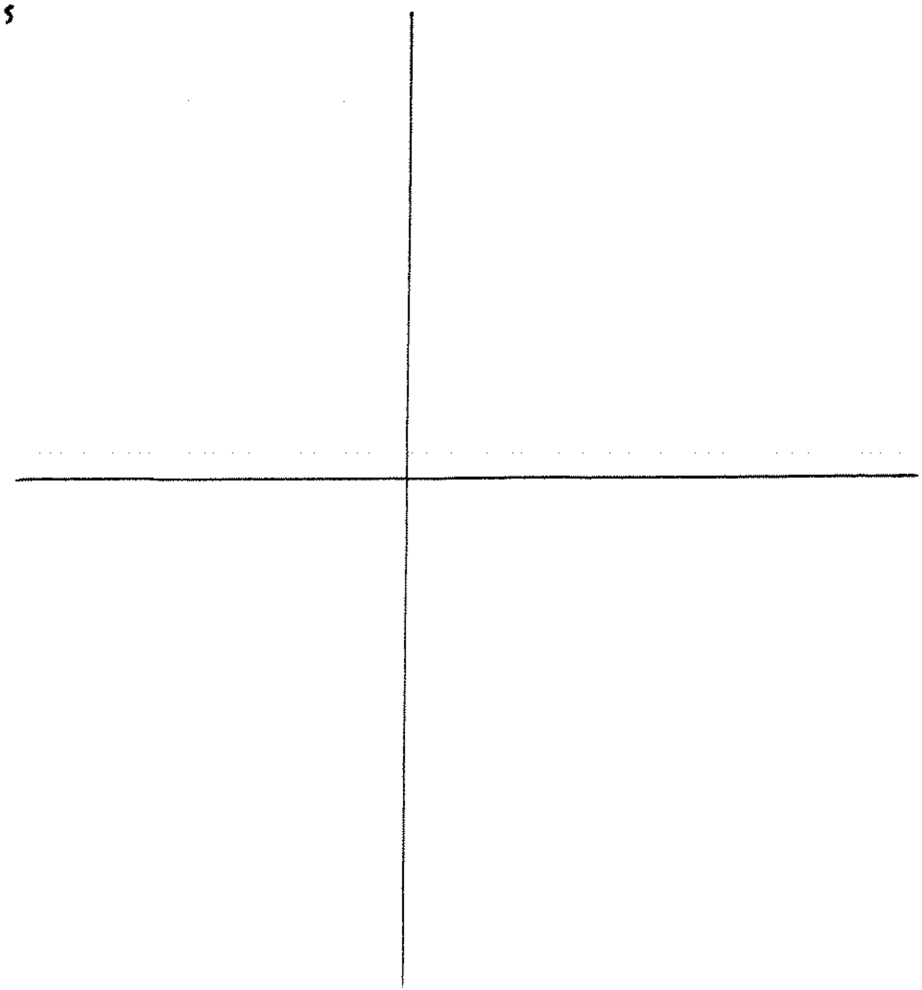
Identify

inc/dec intervals

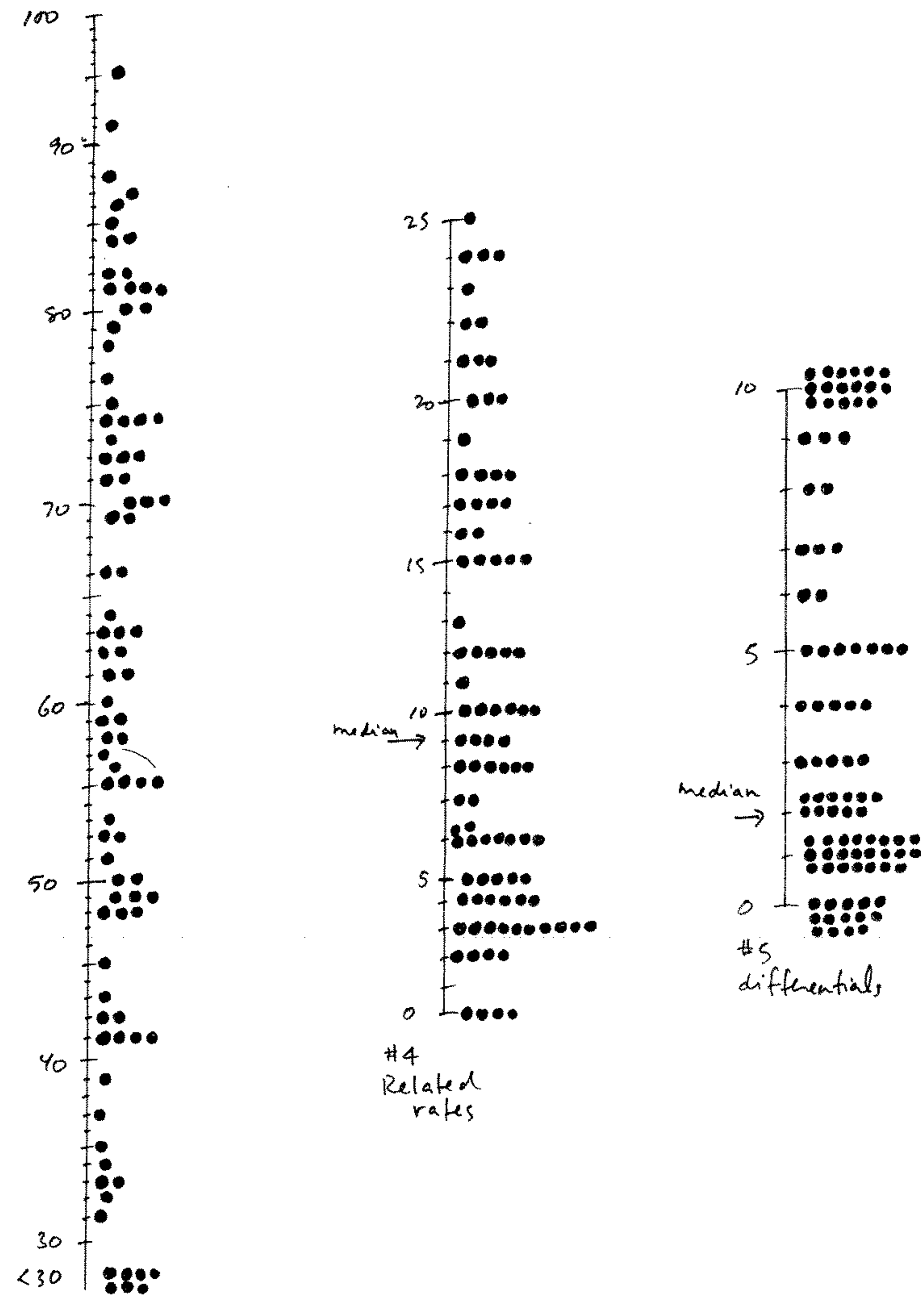
CU/CD intervals

critical pts

local extrema



Exam 2
n=91



100
90
80
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60
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40
30
<30
25
20
15
10
median
5
0
#4
Related
rates
10
5
median
0
#5
differentials